

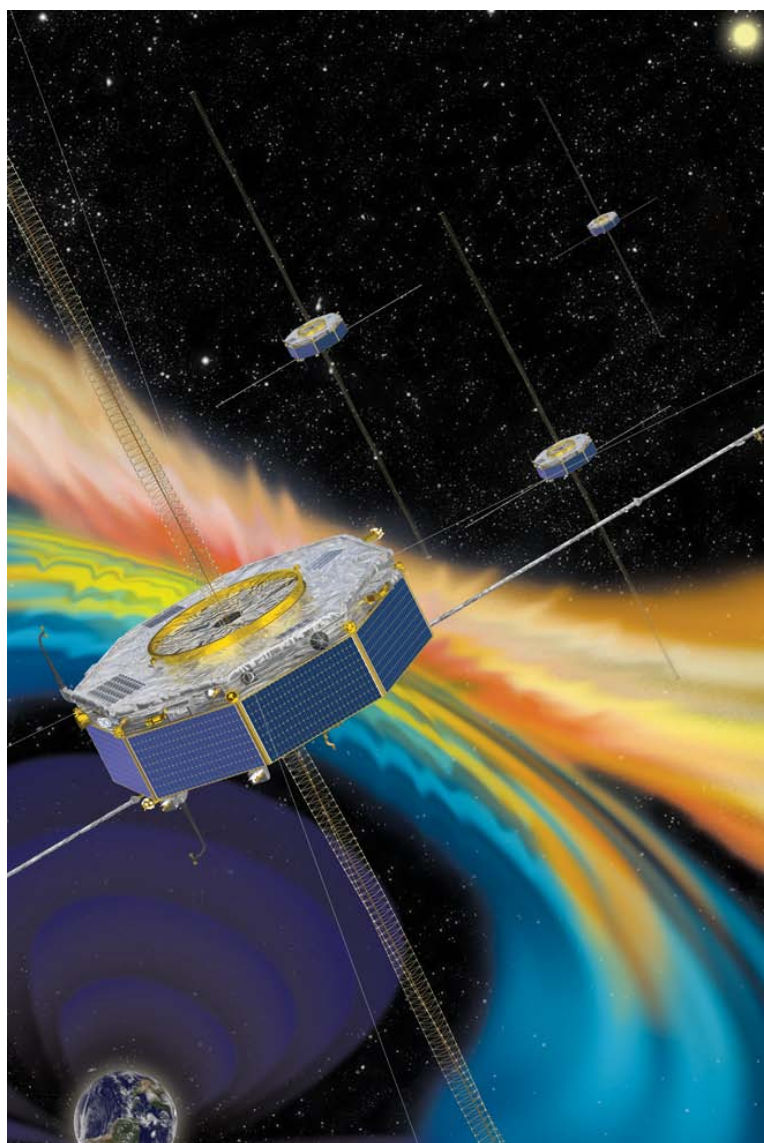


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# *Magnetospheric Multiscale (MMS) Mission*

## *EDUCATOR'S INSTRUCTIONAL GUIDE*

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Educational Products	
Educators & Students	Grades 5-8

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## Program Overview

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### About this Guide:

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This guide uses examples from the MMS Mission to introduce mathematics in a real-world context to fifth through eighth graders. The main area of mathematics covered in this guide is geometry. The guide focuses on two-dimensional and three-dimensional **geometry** to assist students in developing **spatial skills**. There are also some activities that involve algebra and computational skills. The guide contains four lessons that cover multiple mathematics topics and address many National Council of Teachers of Mathematics (NCTM) and Common Core mathematics standards (see Table 1). Each lesson is self-contained and can be used alone or all four can be used together as a unit. If the lessons are used together as a unit the ultimate goal would be to have the students complete a final report which contains a section about each lesson and/or a final project of the students' choice described at the end of the guide.

The guide is meant to help students learn about math utilizing the NASA MMS mission and be able to produce artifacts that can be shared with their peers and with their families. If all four lessons and the Project Summary are completed then the students will have an excellent means of sharing information about the MMS mission and a portfolio of work that can be used to show the students' abilities in several areas including mathematics, literacy, communication, collaboration, creativity, and self-direction.



A variety of supplementary math activities related to the MMS mission are also available in this Guide through a collaboration with **SpaceMath@NASA**. The problems serve as extensions to the primary Guide activities, and are appropriate for more advanced students. Students will be asked to perform specific calculations, solve simple equations, work with areas, percentages and basic concepts in rocket science. For additional mathematics resources visit <http://spacemath.gsfc.nasa.gov>

### Modalities for Learning:

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Every attempt has been made throughout the lessons to address all types of learners including:

**Auditory:** This learner does best by listening and responds to verbal instructions. They solve problems by talking them out.

**Visual:** This learner does best through demonstrations and descriptions. They often make lists or drawings to develop solutions. They have well developed imaginations.

**Tactile:** This learner does well with projects or demonstrations. They like hands-on. They need to take notes when learning something new.

**Kinesthetic:** This learner does best when they are actively involved. They learn best by doing and often have problems sitting still and lose much of what is said or read.

### Overarching Concept:

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The mathematics related to the MMS Mission is the basis for this learning guide. This guide focuses on some of the mathematics related aspects of the MMS Mission. The students will first explore how the satellites are designed and built. They will then explore how the satellites get into their orbit for the mission. The satellite flight configuration during the mission will then be examined. Finally, the students will investigate how the satellites are powered using solar panels.

### Sub Concepts:

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1. Investigate the shape of the MMS satellites by building a model of the satellite.
2. Discover the type of launch vehicles used to carry the satellites into their orbit.
3. Examine the flight configuration of the MMS satellites as they complete their mission.
4. Determine how satellites are they powered while in space.

### Resources:

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All online resources for these lessons can be found on the Magnetospheric Multiscale Mission Main Website related to the instructional guide

[http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)

## 5E Learning Cycle Lesson Plan

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Each of the four lessons utilize the 5E learning cycle. These 5E lessons ask students to use inquiry skills by exhibiting *what they know* and *what they can do* with their knowledge in a real time dimension. Students will construct knowledge and assign meaning to what they have learned and experienced.

### Engage:

*In this section the teacher creates interest, generates curiosity, raises questions and elicits responses that uncover what the students know or think about the topic. The students first encounter and identify the instructional task. They make connections between past and present learning experiences, lay the organizational ground work for the activities ahead and stimulate their involvement in the anticipation of these activities.*

**Explore:**

*In this section the students have the opportunity to get directly involved with the content and materials. Involving themselves in these activities they develop a grounding of experience with the content. As they work together in teams, students build a base of common experience which assists them in the process of sharing and communicating. The teacher encourages the students to work together with minimum supervision, observes and listens to the students, asks probing questions to redirect the students' investigations when necessary. The teacher provides time for students to work through problems, and acts as a facilitator.*

**Explain:**

*In this section the student begins to put the abstract experience through which she/he has gone /into a communicable form. Language provides motivation for sequencing events into a logical format. Communication occurs between peers, the facilitator, or within the learner himself. Working in groups, learners support each other's understanding as they articulate their observations, ideas, questions and hypotheses. The teacher encourages the students to explain concepts and definitions, asks for justification (evidence) and clarification from students, formally provides definitions, explanations, and new labels, and uses students' previous experiences as the basis for explaining new concepts.*

**Elaborate:**

*In this section the students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them. The teacher expects the students to use formal labels, definitions, and an explanation provided previously, and encourages the students to apply or extend the concepts and skills in new situations. The teacher reminds students of the existing evidence and data and asks:*

- *What do you already know?*
- *Why do you think . . .*

**Evaluate:**

*This is an on-going diagnostic process that allows the teacher to determine if the learner has attained understanding of concepts and knowledge. Evaluation and assessment can occur at all points along the continuum of the instructional process.*

## **Instructional Objectives:**

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The students will be able to

- Build a three dimensional scale paper model of one of the MMS satellites.
- Calculate the octagonal area of the top and bottom of the satellite, given the measurements from the satellite.
- Compare the octagonal cross section area of the satellite with the circular cross section area of the launch vehicle to determine if the space craft will fit the cargo bay.

## National Aeronautics and Space Administration

- Compute the speed of the launch rocket, given a data chart of time vs. distance data from lift-off.
- Visualize the three dimensional tetrahedral flight configuration of the four satellites in the mission using graphing techniques and models,
- Analyze, by graphing techniques, the changing shape of the tetrahedron as the satellites change position.
- Compute the volume of the tetrahedron based on the positions of the four satellites.
- Calculate the surface area of the panels that are exposed to the sun for various positions of the satellite, given the dimensions of the solar panels,
- Calculate the power generated by the solar panels for various positions of the satellite, given the dimensions of the panels.
- Organize and write a report about the satellites in the MMS Mission that contains information from the four lessons, after completing the exercises for the unit.

## Lesson 1: Model of the MMS Satellite

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This lesson will allow students to build a scale model of the MMS *satellite* and investigate the geometric properties.

### Objectives:

---

- Build a three dimensional scale paper model of one of the MMS satellites.
- Calculate the octagonal area of the top and bottom of the satellite, given the measurements from the satellite.
- Compare the octagonal cross section area of the satellite with the circular cross section area of the launch vehicle to determine if the space craft will fit the cargo bay.

### General Classroom Requirements:

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#### *Classroom Space Requirements:*

The first lesson requires the students be able to work in groups to construct a model of the MMS Satellite. It will be better to have a room with tables if possible, or move desks in a circle.

**Computer/Internet Station Requirements:** The lesson requires students to access websites. This can be done individually, in groups of two or three or together as a class with a computer and projector. A whole class activity using a computer and a projector is recommended for the engage part of the lesson. An alternative would be to allow each student or group of students to explore the websites on their own and fill out a K-W-L chart as a group. For the explain part of the lesson, it is recommended that the students either have a computer or work in pairs on a computer to write their report.

### Time:

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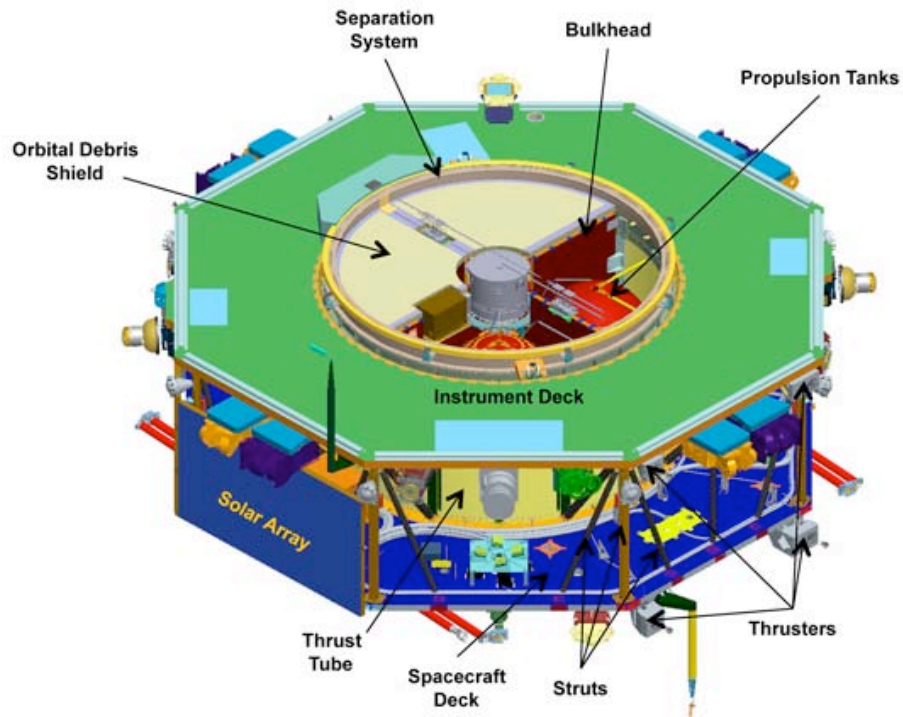
Engage Activity: 15 minutes  
Explore Activity: 30 minutes  
Explain Activity: 45 minutes  
Elaborate Activity: 30 minutes  
Evaluate Activity: 15 minutes  
Total Time: 2 hours 15 minutes (Can break activities over multiple class sessions)

### Content Background:

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Each satellite has an octagonal shape that is approximately 3.5 meters wide and 1.2 meters high. The satellites spin at 3 **revolutions** per minute (RPM) during science operations. There are eight **deployable booms** per satellite: four 60 meter wire booms in the **spin plane** for electric field sensors, two 12.5 meter antennae booms in the **axial plane** for electric field sensors, and two 5 meter antennae booms in the spin plane for **magnetometers**.





More information can be found at Websites for Lesson 1  
[http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)

## Lesson Plan

### ***Engage*** (15 Minutes)

For this part of the lesson, the students will learn about how satellites are designed and built.

#### **Materials:**

Computer

Projector or Smart Board

K-W-L charts on white board

Websites for Lesson 1 [http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)

#### **The Activity: MMS Mission Pre-assessment**

Get started by going to the MMS Mission website for teachers and students (link above) to explore what the mission is all about and for this lesson focus on how the satellites are built and their geometric shape and properties.

Work as a class to complete a “K-W-L” chart (Appendix A) to explore “What We Know”, “What We Want to Know”, and “What We Learned”. This will help students focus on and share what they already know about a subject. You, as the

teacher, will become aware of the general knowledge basis that different students possess, and will be alerted to possible misconceptions that your students may have about particular topics.

One student can act as recorder and can compile a K-W-L chart for the class using the topics, “**What we know about how satellites are designed and built**” and “**What we wonder about the satellites being deployed to collect scientific data?**”

Complete all the lesson activities before completing the last section, “**What we learned about the size and shape of the MMS satellites.**”

Comprehension

Knowledge

***Explore***  
(30 Minutes)

In this activity students will work in groups of four, each student building a model of the satellite. There are three types of models that can be built, paper, edible, or Lego. Each group of four students will represent the four satellites that will be launched into orbit together.

**Materials:**

<b>Paper Model</b>	<b>Edible Model</b>	<b>Model of Lego Bricks</b>
Flat paper model of satellite (Appendix B)	Graham Crackers	Lego Kit
Tape	Frosting	Plans
Colored Pencils or Crayons	Hershey’s Chocolate bars	
Scissors	Licorice whips	
Ruler	Assorted small candies	
String, Thin Wire (Florist Wire) or Coffee Stirrers		

**The Activity:**

The students will work in groups of four. Each student will build a model of one of the MMS Satellites and explore several aspects of the design including:

- the octagonal cross section
- the length of the antennas
- the placement of the satellites into the rocket for launching

They will also attach string (or licorice) that represents the length of the antennas to get a sense of scale. They will determine how large a rocket would have to be to launch four of the satellites. Follow the directions in Appendix B to build a model of the satellite.

Evaluate

Synthesis

Analysis

**Explain**  
(45 Minutes)

**Materials:**

Activity 1: Computing the Area of the Top and Bottom of the MMS Satellite (Appendix C)

**The Activity:**

Students should complete the activity for computing the area of the octagonal top and bottom of the satellite. Students draw labeled diagrams that show how they computed the area of the top of the satellite and how it can fit into the circular hull of the launch vehicle.

*Analysis*

*Application*

*Comprehension*

*Knowledge*

**Elaborate**  
(30 Minutes)

The students will write a paragraph to add to a report about the shape of the satellites and how they are to fit into the circular rocket pay load bay.

**Materials:**

Computer for each student or student team (not necessary)

**The Activity:**

The students will create a report that outlines how the octagonal satellites are placed into the circular cargo bay of the launch vehicle.

Things to include in your report:

- Explain the shape of the MMS satellite
- Explain how the octagonal satellites fit into the circular cargo bay (draw pictures to illustrate)
- Explain how large the antennae are on the MMS satellite

*Evaluate*

*Application*

*Comprehension*

*Synthesis*

**Evaluate**  
(15 Minutes)

**The Activity:**

Encourage students to complete their own report about the MMS satellite to summarize what they now know about the satellite and present them. The teacher will circulate while the students are creating their reports giving help if necessary.

**Sample rubric for grading lesson – Teacher should modify**

	Model	Activity 1	Report	Points Received
Accuracy				/15
Effort Regarding Facts				/15
Effort Regarding Correct Grammar, Punctuation and Sentence Structure				/15
Following Instructions				/15
<b>Total Points:</b>				/60

*Evaluate*

*Synthesis*

*Knowledge*

**Extension Activity:**

There are three versions of models made of Lego bricks available, one that includes the instruments on the satellite. [http://mms.gsfc.nasa.gov/epo\\_mms\\_lego\\_model.html](http://mms.gsfc.nasa.gov/epo_mms_lego_model.html)



Try the **SpaceMath@NASA Supplementary Problem #4 The Volume and Surface Area of an Octagonal MMS Satellite** at the end of this Guide.

## Lesson 2: Launch of the Satellites

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This lesson will give students an overview of the launch vehicle that will take the MMS satellites into space. They will look at some sample launch data and use algebra to compute the speed of the rocket.

### Objectives

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- Compute the speed of the launch rocket, given a data chart of time vs. distance data from lift-off.

### General Classroom Requirements:

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**Classroom Space Requirements:** No additional requirements.

**Computer/Internet Station Requirements:** The lesson requires students to access websites. This can be done individually, in groups of two or three or together as a class with a computer and projector. A whole class activity using a computer and a projector is recommended for the engage part of the lesson. An alternative would be to allow each student or group of students to explore the websites on their own and fill out a K-W-L chart as a group. For the explain part of the lesson, it is recommended that the students either have a computer or work in pairs on a computer to write their report.

### Time:

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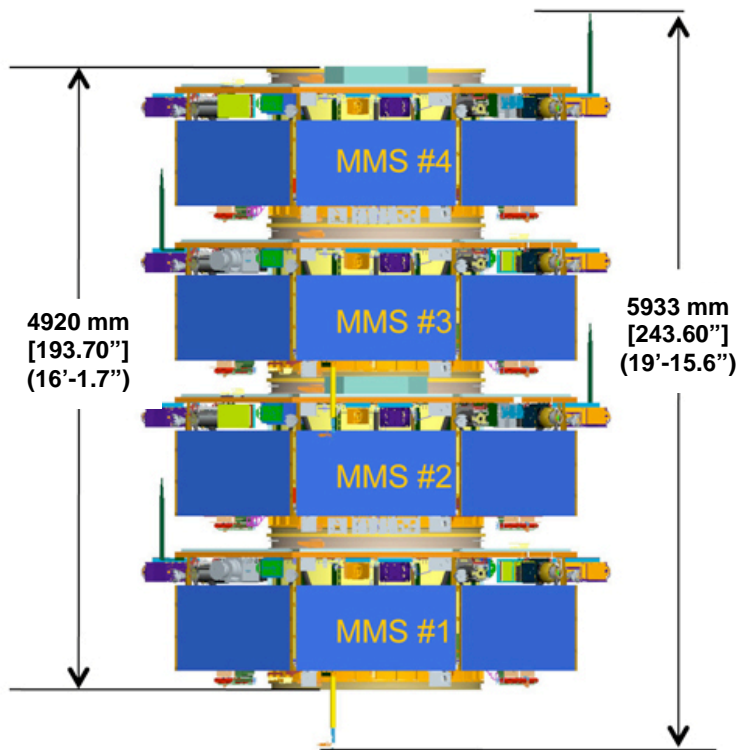
Engage Activity: 15 minutes  
Explore Activity: 30 minutes  
Explain Activity: 45 minutes  
Elaborate Activity: 30 minutes  
Evaluate Activity: 15 minutes  
Total Time: 2 hours 15 minutes (Can break activities over multiple class sessions)

### Content Background:

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The Atlas V 421 Rocket is the vehicle that will be used to take the MMS satellites into space. The Atlas V was developed as part of the US Air Force Evolved Expendable Launch Vehicle (EELV) program. The term expendable launch vehicle means each vehicle is only used once. The MMS satellites will launch from Cape Canaveral Air Force Station in Florida.



The Atlas **rockets** are dedicated to launching certain classes of satellite cargo into **orbit**. The Atlas V 421 that will be used for the MMS Mission has a 4 meter diameter payload bay and has two strap-on solid rocket boosters. The solid rocket boosters are used to provide thrust in spacecraft launches from the launch pad. The four MMS satellites will be stacked in the Atlas V rocket **payload bay** as pictured here.

More facts about the rockets can be found at Websites for Lesson 2 at

[http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)

## Lesson Plan:

### ***Engage*** (15 minutes)

For this part of the lesson, the students will learn about how rockets are used to launch **spacecrafts**, like satellites or **planetary probes**, into space.

#### **Materials:**

Computer  
Projector or Smart Board  
K-W-L charts on white board  
Websites for Lesson 2 [http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)

#### **The Activity: MMS Launch Pre-assessment**

Watch the launch of the rocket that can be found at the website listed above and discuss how the rockets take payloads into space.

Work as a class to complete a “K-W-L” chart to explore “What We Know”, “What We Want to Know”, and “What We Learned”. One student can act as recorder for the class to compile a K-W-L chart using the specific topics, “**What we know about how rockets are launched**” and “**What we wonder about how rockets are used to take things into space?**”

Complete all the lesson activities before completing the last section, “**What we learned about how the MMS satellites will be deployed into space using a rocket.**”

Comprehension

Knowledge

**Explore**  
(30 minutes)

In this activity students will work in groups to collect facts about the specific rocket that will take the MMS satellites into space.

**Materials:**

Websites for Lesson 2 [http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)  
Rocket Worksheet (Appendix D)

**The Activity:**

The students will visit the websites and collect facts about the Atlas V 421 launch rocket for the MMS Mission.

Synthesis

**Explain**  
(45 minutes)

Students will complete an activity on the speed of the Atlas V rocket.

**Materials:**

Activity 2 – Computing the Speed of the Launch Rocket (Appendix E)

**The Activity:**

Students should complete the activity for computing the speed of the rocket at different time intervals. They will predict speeds using trending from the data given.

Evaluate

Analysis

**Elaborate**  
(30 Minutes)

The students will write a paragraph to add to a report about the rocket that will take the MMS satellites into space.

**Materials:**

Computer for each student or student team (not necessary)

**The Activity:**

The students will create a report that outlines how large the rocket payload bay is and how the satellites fit in there. They should also describe how a rocket will be used to take the payload into space and how fast the rocket is going.

Things to include in your report:

- Explain where the satellites are placed in the rocket.
- Explain what the rocket uses for fuel.
- Explain how fast the rocket will be going when it is at certain distances from the earth.
- Other facts from the Rocket Fact Sheet

The students will write a report about the launch of the rocket to take the MMS satellites into space.

*Evaluate*

*Synthesis*

*Application*

*Comprehension*

**Evaluate**  
(15 Minutes)

**The Activity:**

Encourage students to complete their own report about the MMS satellite launch to summarize what they learned about the rocket and present them. The teacher will circulate while the students are creating their reports giving help if necessary.

**Sample Rubric for grading lesson** – Teacher should modify

	Activity 1	Report	Points Received
Accuracy			/15
Effort Regarding Facts			/15
Effort Regarding Correct Grammar, Punctuation and Sentence Structure			/15
Following Instructions			/15
<b>Total Points:</b>			/60

*Evaluate*

*Synthesis*

*Knowledge*





**Extension Activity:**

Have students construct a model of the rocket payload bay using things like poster board or large paper, which the satellites from Lesson 1 will fit into. Have groups of four students stack the satellites for launch inside their rocket payload bay.



Try the **SpaceMath@NASA Supplemental Problems**

- #5 Some Statistical Facts about the MMS Atlas Rocket,**
- #6 Launching the MMS Satellite Constellation Into Orbit,**
- #7 The Magnetosphere Multi-Scale Payload – Up Close,**
- #8 Exploring the Atlas – V Launch Pad at Cape Canaveral and,**
- # 9 The Acceleration Curve for the Atlas V MMS Launch**  
at the end of this Guide.

## Lesson 3: The Satellites Flight Configuration

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This Lesson will give students an overview of the how the MMS satellites fly through space to carry out their mission.

### Objectives:

---

- Visualize the three dimensional tetrahedral flight configuration of the four satellites in the mission using graphing techniques and models,
- Analyze, by graphing techniques, the changing shape of the tetrahedron as the satellites change position.
- Compute the volume of the tetrahedron based on the positions of the four satellites.

### General Classroom Requirements:

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*Classroom Space Requirements:* No additional requirements.

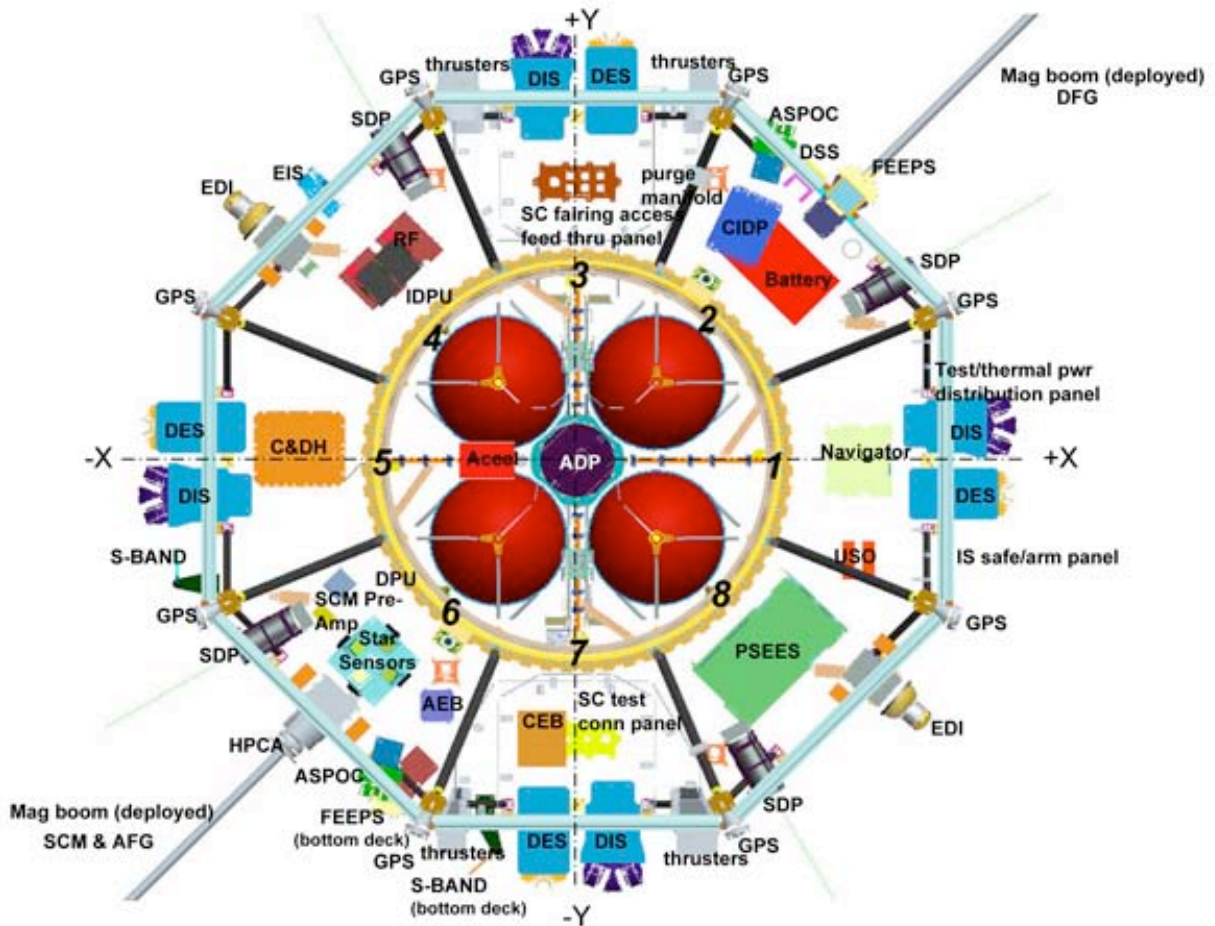
*Computer/Internet Station Requirements:* The lesson requires students to access websites. This can be done individually, in groups of two or three or together as a class with a computer and projector. A whole class activity using a computer and a projector is recommended for the engage part of the lesson. An alternative would be to allow each student or group of students to explore the websites on their own and fill out a K-W-L chart as a group. For the explain part of the lesson, it is recommended that the students either have a computer or work in pairs on a computer to write their report.

### Time:

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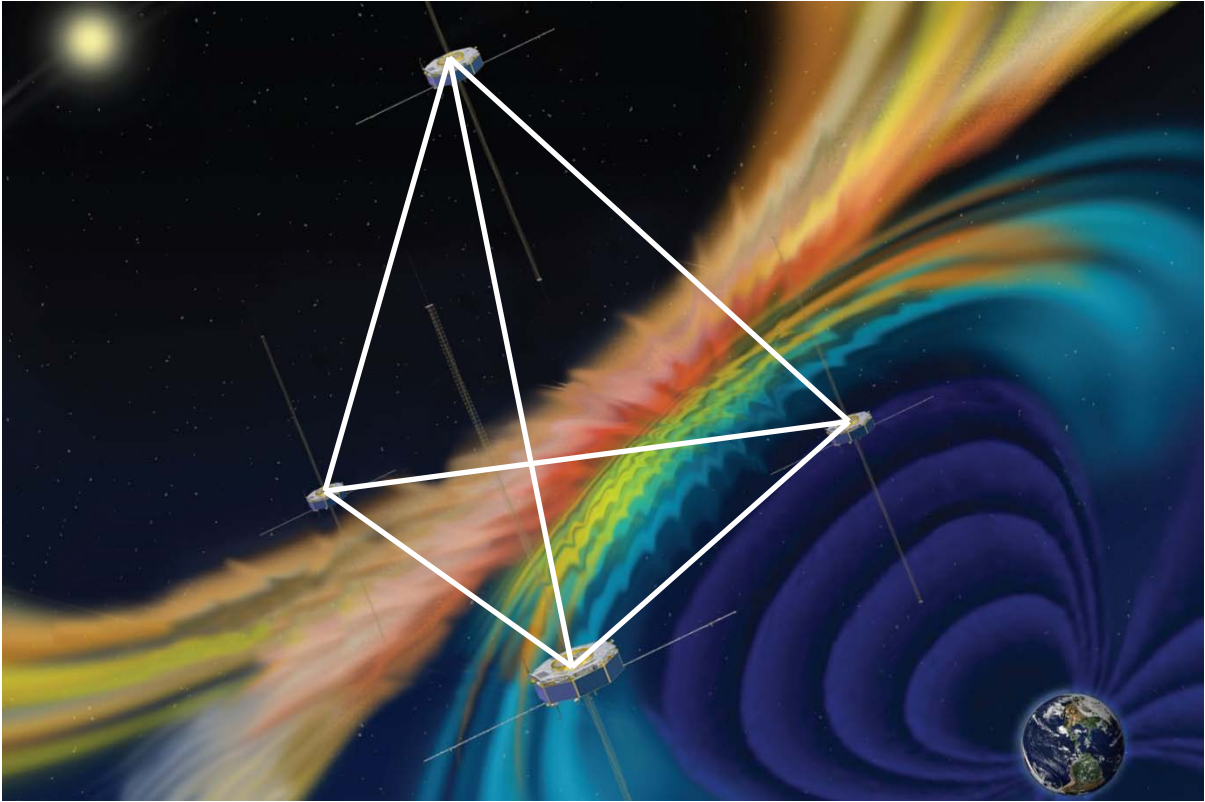
Engage Activity: 15 minutes  
Explore Activity: 30 minutes  
Explain Activity: 45 minutes  
Elaborate: 30 minutes  
Evaluate Activity: 15 minutes  
Total Time: 2 hours 15 minutes (Can break activities over multiple class sessions)

## Content Background:



The MMS satellites, once they leave the launch vehicle fly in formation. The satellites have a **propulsion** system with 12 **thrusters**, sized to achieve both small formation maintenance maneuvers and larger maneuvers that move them into different orbits. The satellites' propellant and something called a mono-propellant blowdown system help them maintain their formation during their long mission through space. The propellant to help the satellites get into the correct position is contained in four titanium tanks per spacecraft.

The satellite flight configuration is a tetrahedron, with the four satellites as the four vertices of the tetrahedron. A tetrahedron is a polyhedron composed of four triangular faces. The tetrahedron is one kind of pyramid. In a regular tetrahedron, all four of the triangular faces are equilateral triangles (equal sides and equal angles). During the MMS mission, the shape of the tetrahedron changes as the satellites change positions.



The volume of a tetrahedron is given by the pyramid volume formula:

$$V = \frac{1}{3}Bh$$

where  $B$  is the area of the base and  $h$  the height from the base to the **apex**. This applies for each of the four choices of the base.

## Lesson Plan:

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### ***Engage*** **(15 Minutes)**

#### **MMS Flight Configuration Pre-assessment**

For this part of the lesson, the students will watch a video of the orbit and formation of the MMS satellites as they fly through space and learn about satellite flight configurations.

#### **Materials:**

Computer

Projector or Smart Board

K-W-L charts on white board

Websites for Lesson 3 [http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)

#### **The Activity: MMS Flight Configuration Pre-assessment**

Watch a visualization of the MMS satellite's orbit and how the satellites fly in tetrahedral formation in the video on the website listed above.

“The movie initially shows the general orientation of the orbit with respect to the Earth, Moon, and Sun. It then zooms in to “ride” along with the spacecraft. We then zoom in even closer to show that there are actually four spacecraft flying in a tetrahedral formation.<sup>1</sup>”

Watch the video of the satellites flying in formation. One student can act as recorder and can compile a KWL chart for the class using the topics, “**What we know about how satellites fly in formation**” and “**What we wonder about the MMS satellites’ formation**”

Complete all the lesson activities before completing the last section, “**What we learned about how the MMS satellites fly in a three dimensional formation to accomplish their mission.**”

*Comprehension*

*Knowledge*

---

<sup>1</sup> [http://mms.gsfc.nasa.gov/videos\\_animations.html](http://mms.gsfc.nasa.gov/videos_animations.html)

**Explore**  
(30 minutes)

In this activity students will work in groups to collect facts about the other types of satellites and satellite configurations in space.

**Materials:**

Websites for Lesson 3  
Formation Flying Worksheet (Appendix F)

**The Activity:**

The students will visit websites and collect facts about different types of satellites that fly in formation.

**Synthesis**

**Explain**  
(45 minutes)

**Materials:**

Activity 3: Computing the Volume of the Flight Configuration (Appendix G)  
Tape  
Dot paper

**The Activity:**

Students will compute the volume of the tetrahedron that the four satellites make as they fly in their flight configuration. They will investigate how the volume of the tetrahedron changes as the satellites change positions.

**Evaluate**

**Synthesis**

**Analysis**

**Elaborate**  
(30 Minutes)

The students will write a section for their report about the flying formation of the MMS satellites in space.

**Materials:**

Computer for each student or student team (not necessary)

**The Activity:**

The students will create a report that outlines how the MMS satellites fly in a tetrahedral formation through space.

Things to include in your report:

- Give some examples of different satellites and explain how the different satellites fly in formation
- Explain the shape of the MMS satellite flight formation from the information you gained from the video and your activity
- Explain how the satellites can change position in space
- Explain how the volume of the flight configuration shape changes when the satellites change position (draw pictures to illustrate)



**Evaluate**  
(15 Minutes)

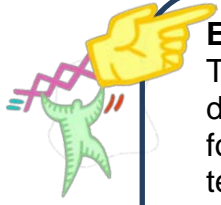
**The Activity:**

Encourage students to complete their own report about the MMS satellite flight configuration. The teacher will circulate while the students are creating their reports giving help if necessary.

**Sample Rubric for grading lesson** – Teacher should modify

	Activity 1	Report	Points Received
Accuracy			/15
Effort Regarding Facts			/15
Effort Regarding Correct Grammar, Punctuation and Sentence Structure			/15
Following Instructions			/15
<b>Total Points:</b>			<b>/60</b>





**Extension Activity:**

Teams of four students could use the scale models they built for Lesson 1 and use dowel rods or coat hanger wire to assemble these scale models into a tetrahedral formation and hang in the classroom like a mobile. Students could also build a tetrahedral kite, have them look for instructions for this on the internet.



Try the **SpaceMath@NASA Supplementary Problem #1 The Orbit of the MMS Satellite Constellation** and **#2 The Orbit of the MMS Satellite Constellation** at the end of this Guide.



## **Lesson 4: Powering the Satellite (Solar Panels)**

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This lesson will give students an understanding of how much power can be generated by the solar panels on the MMS Satellite.

### **Objectives:**

---

- Calculate the surface area of the panels that are exposed to the sun for various positions of the satellite, given the dimensions of the solar panels,
- Calculate the power generated by the solar panels for various positions of the satellite, given the dimensions of the panels.
- Organize and write a report about the satellites in the MMS Mission that contains information from the four lessons, after completing the exercises for the unit.

### **General Classroom Requirements:**

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*Classroom Space Requirements:* No additional requirements.

*Computer/Internet Station Requirements:* The lesson requires students to access websites. This can be done individually, in groups of two or three or together as a class with a computer and projector. A whole class activity using a computer and a projector is recommended for the engage part of the lesson. An alternative would be to allow each student or group of students to explore the websites on their own and fill out a K-W-L chart as a group. For the explain part of the lesson, it is recommended that the students either have a computer or work in pairs on a computer to write their report.

### **Time:**

---

Engage Activity: 15 minutes  
Explore Activity: 30 minutes  
Explain Activity: 30 minutes  
Elaborate Activity: 30 minutes  
Evaluate Activity: 15 minutes  
Total Time: 2 hours (Can break activities over multiple class sessions)

## Content Background:

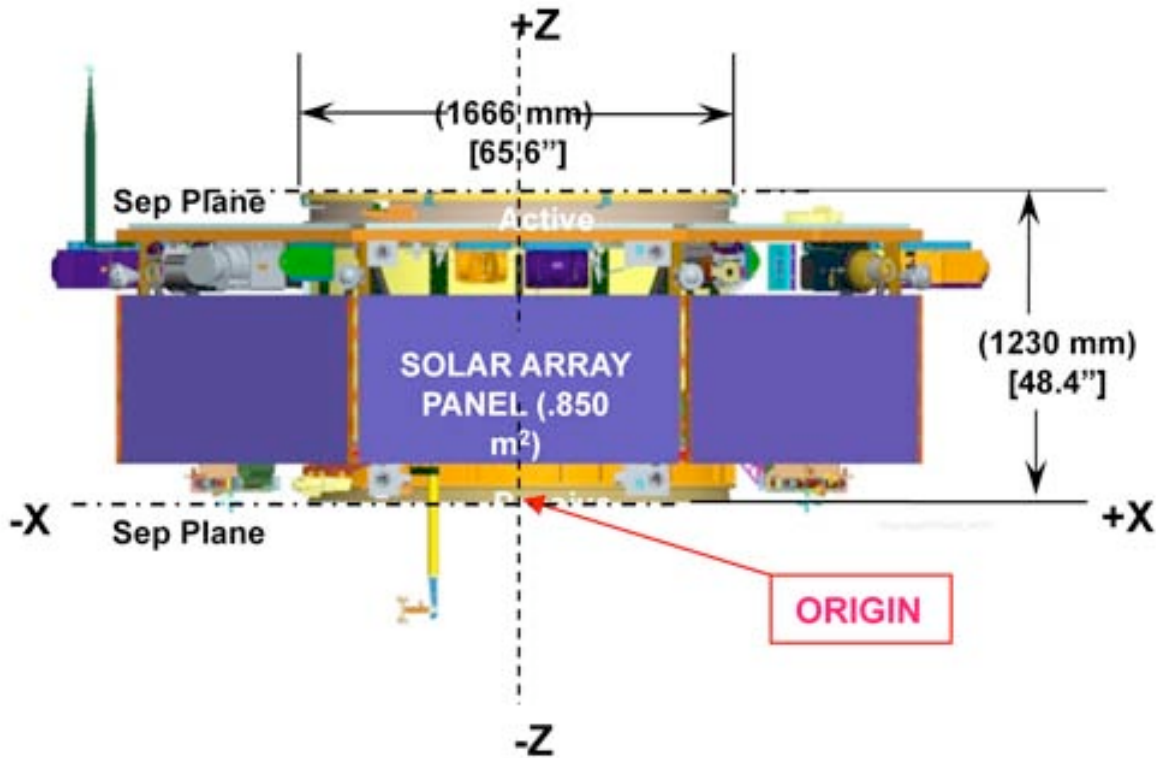


Figure 1: Satellite Solar Array Panel

Electrical power is needed to run many of the instruments and systems on the satellites. The satellites use **solar energy** to generate electricity to run their systems. The electrical power on each MMS Satellite is supplied from eight identical body-mounted solar array panels. Some of the power is also stored in on-board batteries. The batteries are sized to provide power during the four hour **eclipses** where the satellite would not be exposed to the Sun.

The table below shows some common appliances around your house and how much electricity they take to use. As you do the activities to see how much power can be produced by the solar panels, you can see in the table how much each of these would take to run by comparison.

Table 1: Example Power Usage Chart

Power Usage Charts for Appliances	Running Watts
Refrigerator	500
Microwave Oven	1200
Television	500
Personal Computer	700
Fan	250

## Lesson Plan:

---

### **Engage** (15 minutes)

For this part of the lesson, the students will learn how solar panels can convert sun light into electricity.

**Materials:**

Computer

Projector or Smart Board

K-W-L charts on white board

Websites for Lesson 4 [http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)

**The Activity: MMSSolar Power Pre-Assessment**

Look at the NASA web page above that explains how light is converted into electricity. One student can act as recorder and can compile a K-W-L chart for the class using the topics, “**What we know about solar power**” and “**What we wonder about how a spacecraft could use solar power.**”

Complete all the lesson activities before completing the last section, “**What we learned about how much power is generated by the solar array panels on the MMS satellite.**”

Comprehension

Knowledge

### **Explore** (30 minutes)

In this activity students will work in groups to find examples of other NASA spacecraft and vehicles that use solar power. Have different students or groups of students investigate different NASA mission websites to collect information on the fact sheet. After about 10-15 minutes for gathering information and finding examples, student report on what they or their group found to the rest of the class.

**Materials:**

Spacecraft Solar Panel Worksheet (Appendix H)

Websites for Lesson 4 [http://mms.gsfc.nasa.gov/epo\\_math\\_guide.html](http://mms.gsfc.nasa.gov/epo_math_guide.html)

**The Activity:**

The students will visit the websites and collect facts about the spacecraft that use solar power. If they are using a computer they could bookmark or collect pictures of spacecraft to show different configurations of the solar panels or draw diagrams to show what they saw to share with the class.

Synthesis

**Explain**  
(30 minutes)

**Materials:**

Activity 4 – Computing the Electrical Power Generated by the Solar Panels  
(Appendix I)  
Scissors

**The Activity:**

Students will complete the activity that allows them to find the range of power that will be generated by the solar array panels on each of the MMS Satellites.

*Evaluate*

*Synthesis*

*Analysis*

*Application*

*Comprehension*

**Elaborate**  
(30 Minutes)

The students will write a section for their report about the solar power generated by the MMS satellites in space.

**Materials:**

Computer for each student or student team (not necessary)

**The Activity:**

The students will create a report that outlines how the MMS satellites' solar panels generate power to run the systems of the spacecraft.

Things to include in your report:

- Give examples of different NASA spacecrafts and vehicles that use solar power.
- Discuss the different configurations of solar array panels on the spacecrafts and vehicles.
- Explain how many solar panels a MMS satellite has and how they are located on the spacecraft.
- Explain how many sides can be facing the sun at one time and the range of power that can be generated by each spacecraft.

*Evaluate*

*Synthesis*

*Analysis*

*Application*

*Comprehension*

**Evaluate**  
(15 Minutes)

**The Activity:**

Encourage students to complete their own report about the MMS satellite's solar power. The teacher will circulate while the students are creating their reports giving help if necessary.

**Sample Rubric for grading lesson** – Teacher should modify

	Activity 1	Report	Points Received
Accuracy			/15
Effort Regarding Facts			/15
Effort Regarding Correct Grammar, Punctuation and Sentence Structure			/15
Following Instructions			/15
<b>Total Points:</b>			<b>/60</b>



**Extension Activity:**

Have students look up the types of systems used on the MMS satellite and investigate how much energy each system would take to run.



Try the **SpaceMath@NASA Supplementary Problem #3 MMS Satellites and Solar Power** at the end of this Guide.



## **Project Summary MMS Tic-Tac-Toe**

---

### **Purpose:**

To engage students with the material in a meaningful way to further explore personal interests regarding the MMS project.

### **Activity:**

Students are to choose three activity squares which work to form a straight line diagonally, vertically or horizontally through the center square. The center square, **Student Choice Activity**, is a required task and is developed by the student and completed with prior approval from the teacher.

### **Completed Tic-Tac-Toes Project:**

- Tic-Tac-Toe Cover Sheet  
(Includes student name, date, title, and highlighted project choices.)
- Tic-Tac-Toe Rubric
- Activity 1 Components
- Activity 2 Components
- Activity 3 Components

### **Project Submission:**

There are multiple modes of submission that teachers may choose from for their students to submit their final project. The following is a list of suggested resources and ideas that can be implemented for the submission process.

- Hand-written and/or Computer Printed Documents
- Electronic Files
  - o Google Tools (<http://www.google.com/educators/tools.html> )
  - o Edmodo (<http://www.edmodo.com/> )

### **Grading:**

Student's projects will be graded using the *Magnetospheric Multiscale Mission: Tic-Tac-Toe Project Rubric*. Credit will be awarded based upon a point scale for each of the three activities if completed as it is described. It is the teacher's discretion as to how the student's final project grade will be weighted. (e.g. Some teachers may decide to triple the student's final grade. Student's Points: 40/60 Final Grade: 120/180)

# Magnetospheric Multiscale Mission

## *Tic-Tac-Toe Project Guide*

**Directions:** Choose three activity squares which work to form a straight line diagonally, vertically or horizontally through the center square. The center square, **Student Choice Activity**, is a required task that you would like to complete as a part of your final project. The activity that you choose must have your teacher's prior approval. Using a highlighter or colored pencil please shade the squares denoting the project choices you have made.

<p><b><u>Write</u></b></p> <p>Using a current events article (online or print) regarding satellites write a one-page report summarizing its contents and how it relates to what you have learned while studying about the MMS Mission.</p>	<p><b><u>Develop</u></b></p> <p>Develop an informational brochure outlining the mission.</p>	<p><b><u>Create</u></b></p> <p>Choose a topic relating to satellites and create a poster that accurately represents and explains the topic.</p>
<p><b><u>Discuss</u></b></p> <p>Discuss the purpose of satellites including why they are used to collect scientific data. You may present this information in the form of a slideshow, poster, advertisement, or a written document.</p>	<p><b>Student Choice Activity</b> <i>(Teacher Approval Required)</i></p>	<p><b><u>Construct</u></b></p> <p>Construct a 3-dimensional model of the MMS Satellite using a 3D online tool like Google SketchUp. Label key elements of the satellite.</p>
<p><b><u>Compose</u></b></p> <p>Compose a rap or a song reviewing the events of the MMS Mission.</p>	<p><b><u>Experiment</u></b></p> <p>Using different polygons, experiment to find the relationship between the number of sides and the amount of solar power each would be able to collect. Compare the figures and how much energy each would produce for the satellite, then present your findings in a MS Power Point presentation.</p>	<p><b><u>Generate</u></b></p> <p>A video that tells your friends about the MMS mission and what you have learned. Post this to a place where other students can view it to learn about the MMS mission.</p>

**Student Choice Activity:**

\_\_\_\_\_

**Approval:** \_\_\_\_\_ (Teacher Initials)

# Magnetospheric Multiscale Mission

## *Tic-Tac-Toe Project Rubric*

Name: \_\_\_\_\_

Period: \_\_\_\_\_

Date: \_\_\_\_\_

	<b>5</b>	<b>4</b>	<b>3</b>	<b>2</b>	<b>1</b>
Accuracy	All information is correct. No errors are found.	Most information is correct. Only 1 or 2 errors found.	Some information is correct. Between 3 and 5 errors found.	Little information is correct. 5 or more errors found.	No information is correct.
Content	Content is exceptional and relevant to the topic.	Content is relevant to the topic.	Some content is relative to the topic.	A small amount of content is relative to the topic.	Effort is not relevant to the topic.
Presentation	Presentation is creative and well-organized.	Good effort is made to present information in a neat and creative way.	Some effort was involved in the presentation.	Little effort was involved in the presentation.	No effort was involved in the presentation.
Following Instructions	All instructions are followed completely. No aspect of the activity contradicts the instructions.	Most instructions are followed completely. Only one aspect of the activity contradicts the instructions.	Some instructions are followed completely. Two aspects of the activity contradict the instructions.	Few instructions are followed completely. Three aspects of the activity contradict the instructions.	Instructions were not followed completely. More than three aspects of the activity contradict the instructions.

	<b>Activity 1</b>	<b>Activity 2</b>	<b>Activity 3</b>	<b>Points Received</b>
Accuracy				/15
Effort Regarding Facts				/15
Effort Regarding Presentation				/15
Following Instructions				/15
<b>Total Points:</b>				/60



## Glossary of Terms

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<b><i>apex</i></b>	The tip, point, or vertex.
<b><i>axial plane</i></b>	An imaginary horizontal plane that divides the body into superior and inferior parts.
<b><i>deployable booms</i></b>	A folded structure that can be elongated when a spacecraft is in space. These structures form the building blocks of many larger space apertures, from solar sails to large antenna reflectors.
<b><i>eclipses</i></b>	An astronomical event that occurs when an astronomical object is temporarily obscured, either by passing into the shadow of another body or by having another body pass between it and the viewer.
<b><i>magnetometers</i></b>	A measuring instrument used to measure the strength or direction of magnetic fields.
<b><i>orbit</i></b>	The gravitationally curved path of an object around a point in space.
<b><i>payload bay</i></b>	Area on a spacecraft utilized to carry cargo.
<b><i>planetary probes</i></b>	A spacecraft carrying instruments intended for use in exploration of the physical properties of outer space or celestial bodies other than Earth.
<b><i>propulsion</i></b>	Movement caused by a force.
<b><i>revolutions</i></b>	The time taken by a celestial body to make a complete round in its orbit; the rotation of a celestial body on its axis.
<b><i>rockets</i></b>	A spacecraft that contains thrust from a rocket engine.
<b><i>satellite</i></b>	An object which has been placed into orbit by human endeavor to collect various types of information for research, communication, weather, etc.
<b><i>spacecraft</i></b>	A vehicle, vessel or machine designed for spaceflight.
<b><i>solar energy</i></b>	Radiant light and heat from the sun that has been harnessed by individuals to create an alternative source of power.

***tetrahedron***

A polyhedron composed of four triangular faces, three of which meet at each vertex. It has six edges and four vertices. The tetrahedron is the only convex polyhedron that has four faces.

***thrusters***

A small propulsive device used by spacecraft.

## Appendix A: Template for K-W-L Chart

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In the first column, write what you already **know** about the topic. In the second column, write what you **want** to know about the topic. After you have completed the lesson, write what you **learned** about the topic in the third column.

What I KNOW	What I WANT to Know	What I LEARNED

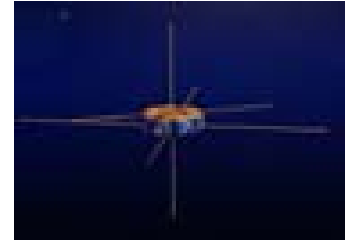
## Appendix B: Directions for Paper Model, Edible Model and Model made of Lego bricks of MMS Satellite

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### 1. Paper Model

You will need

- Paper model (next page)
- Scissors
- Tape
- String or Thin Wire (Florist Wire)

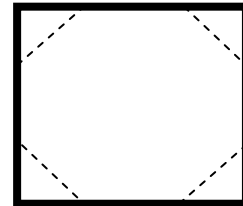


The paper model is at the end of the guide. You will complete the model by cutting out and taping the top and bottom sections together. Make sure that each numbered side on the top section matches the numbered panel on the bottom section. Attach the string or florist wire to the model to simulate the eight deployable booms per satellite.

### 2. Edible Model

You will need

- Graham Crackers
- Frosting
- Chocolate bars with small sections
- Licorice whips
- Assorted small candies



Procedure:

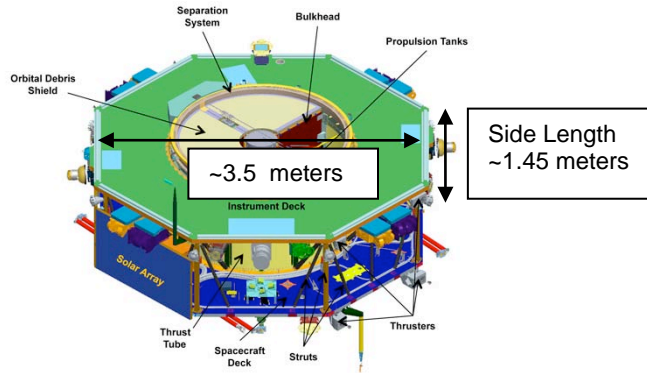
1. Use a square Graham Cracker for the top and bottom of the satellite. To form the octagonal shape cut a small triangle from each corner of the square. This will form an octagon when you have completed all four cuts. Make a top and a bottom for the satellite.
2. Use a generous layer of frosting to attach the top to the bottom. Put enough so that you will be able to add the solar panels to the eight sides.
3. Use the Chocolate bars for the solar panels. Break small rectangles of chocolate to fit on the eight sides.
4. Attach small candies to represent instruments on the sides.
5. Attach licorice whips for the booms.

### 3. Model made of Lego bricks

You can find the instructions for the model made of Lego bricks at the website [http://mms.gsfc.nasa.gov/epo\\_mms\\_lego\\_model.html](http://mms.gsfc.nasa.gov/epo_mms_lego_model.html)

## Appendix C: Worksheet for Computing the Area of the Top and Bottom of the MMS Satellite

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1. Look at the diagram of the satellite above. The shape of the top and bottom is a regular shape (all sides and angles are the same). What is this regular shape? Do not consider the circle in the center, just the outside side of the shape.
2. How can you compute the area of this shape? Do you know a formula or can you devise a strategy for computing the area? Write the formula or describe your strategy for finding the area.
3. Draw the shape of the top of the satellite below and then compute the area of the shape of the top of the satellite using your formula or strategy.
4. The satellite will be sent into space in a rocket with a circular cargo bay. How large would the cargo bay need to be? Calculate the circular radius and circumference that would hold the satellite and draw a diagram of the cross-section below to show how this will fit. Make sure you label your diagram.

## Appendix D: Rocket Worksheet

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Go to the website listed below and find the facts about the rockets (launch vehicles).  
[http://www.ulalaunch.com/site/pages/Education\\_RocketScience.shtml](http://www.ulalaunch.com/site/pages/Education_RocketScience.shtml)

1. Rockets (launch vehicles) deliver \_\_\_\_\_ into space.
2. A typical rocket produces more than \_\_\_\_\_ pounds of thrust.
3. A rocket can carry more than \_\_\_\_\_ pounds.
4. A rocket can go a speed of up to \_\_\_\_\_ miles per hour.
5. The rocket delivers its payload to \_\_\_\_\_.

Go to the website below and find additional facts about the Atlas V rocket, the specific rocket that will take the MMS satellites into space.

[http://www.ulalaunch.com/site/pages/Products\\_AtlasV.shtml](http://www.ulalaunch.com/site/pages/Products_AtlasV.shtml)



6. How many pounds of thrust can the Atlas V engine deliver at liftoff? \_\_\_\_\_

7. What does the Atlas V rocket use for fuel?  
\_\_\_\_\_

8. What is the diameter of the Atlas V 400 series payload fairing which carries the cargo into space?  
\_\_\_\_\_

9. Find the diagram of the Atlas V 400 series that shows the expanded view. From the diagram, where is the cargo bay that carries the satellites?  
\_\_\_\_\_

10. What are the different lengths of the payload fairing for the Atlas V 400 series rockets?  
\_\_\_\_\_

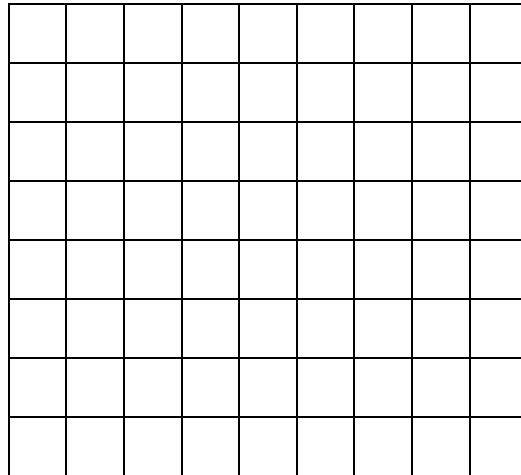
## Appendix E: Worksheet for Computing the Speed of the Launch Rocket

The Atlas V 421 is the launch vehicle that takes the MMS satellites into space. The picture below shows the rocket taking off and the chart to the right shows the approximate height of the rocket at the different times.



Time (sec)	Height (m)	Speed (m/sec)
0	0	0
1	26	
2	58	
3	96	
4	140	

1. Graph Time (sec) vs. Height (m) of the rocket using the graph paper below. Use **Time** as the x-coordinate and **Height** of the rocket as the y-coordinate.

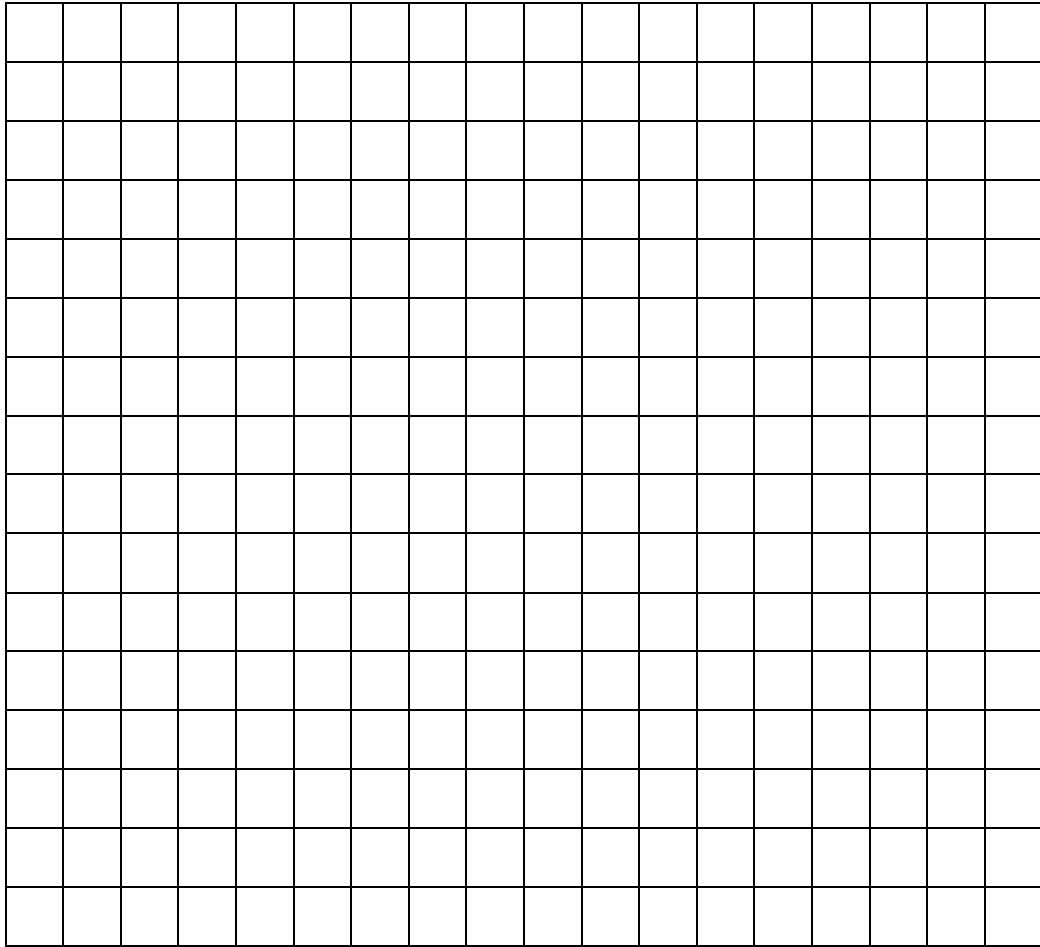


2. Compute the average speed for each time interval and complete the table above. Remember speed has the unit of meters/second. The calculation for average speed in a time interval is determined by

$$\text{Average speed in a time interval} = \frac{\text{Height 2} - \text{Height 1}}{\text{Time 2} - \text{Time 1}}$$

Note: If you have studied the slope of a line this will look familiar to you. You are calculating the slope of the line between two adjacent points.

3. Graph Time vs. Speed of the rocket using the graph paper below. Use **Time** as the x-coordinate and **Speed** as the y-coordinate.
4. Assuming this pattern continues, use your graph to estimate **when** the rocket will be traveling near the speed of 100 meters per second?





## Appendix F: Formation Flying Worksheet

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Explore websites to find facts about how satellites fly in formation.

1. Give an example from nature of birds that fly in formation. \_\_\_\_\_

2. Explain the concept of satellite formation flying.

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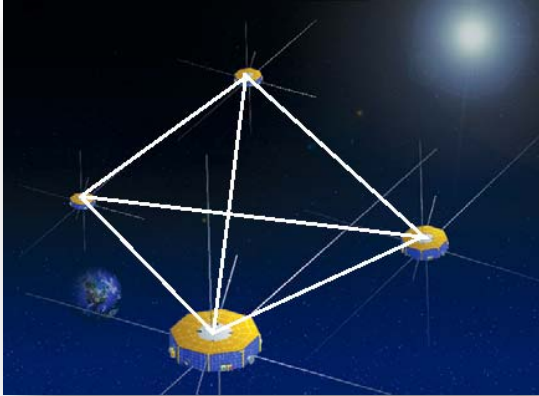
3. What satellites are examples of formation flying?

4. What do these satellites do?

5. Name the satellites that fly in formation.

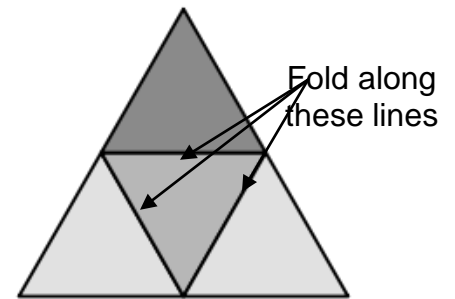
## Appendix G: Directions for Computing the Volume of the Flight Configuration

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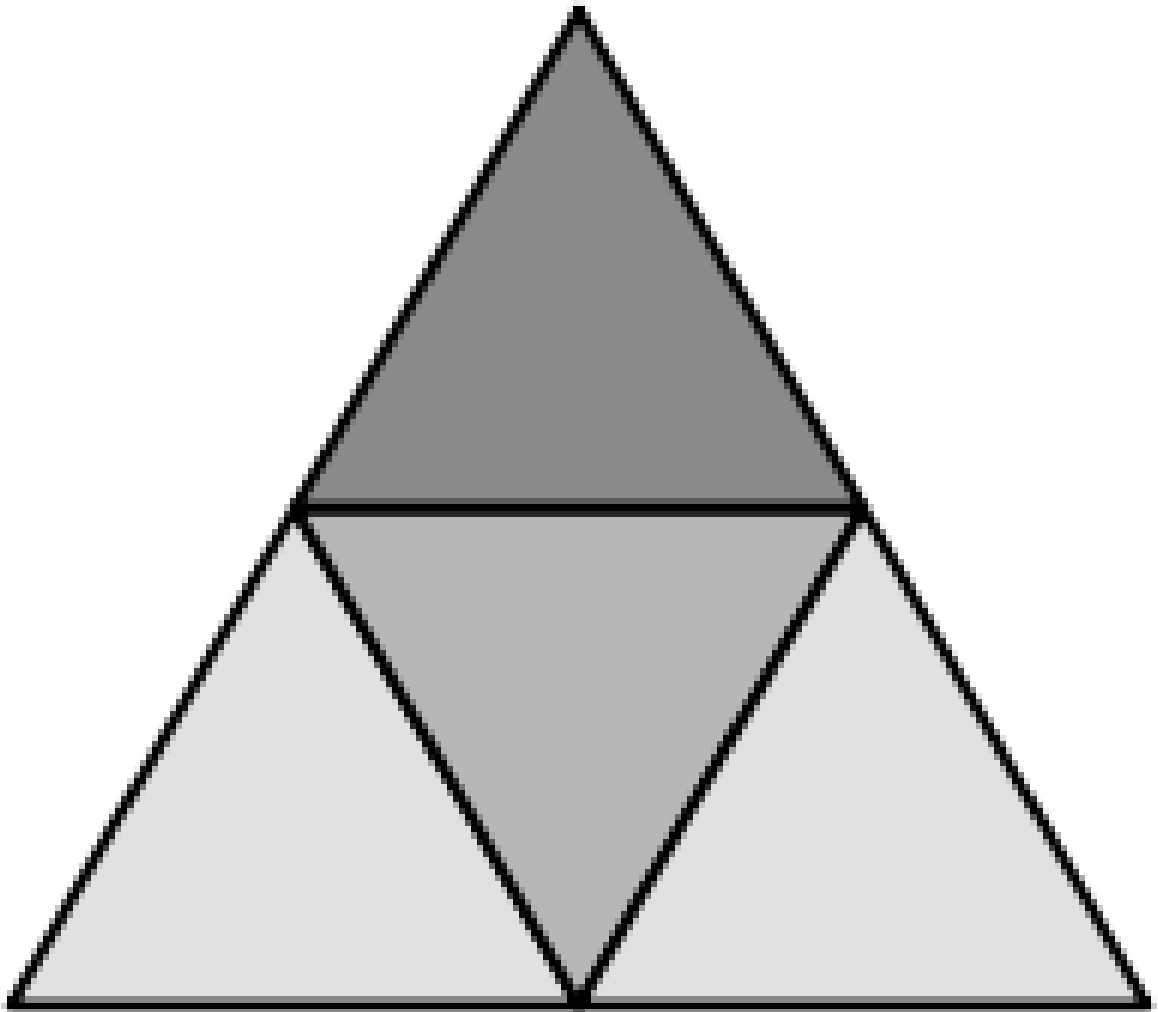
The four MMS satellites form a tetrahedron while flying through space. The picture on the left shows the satellites in their configuration. The satellites change position while traveling through space to change the volume of the tetrahedron they create.

1. On the next page there is a net (two dimensional representation of a three dimensional object) the same as pictured to the right. Cut along the outside of the net and fold forward along the other lines. Assemble the tetrahedron by forming a peak with the three outside points and taping the sides.



2. Use a ruler to measure the side length of the base triangle and the height of the triangle. Using the formula for **area of a triangle**  $A = \frac{1}{2}bh$ . First, compute the area of the base triangle of the tetrahedron (any of the triangles can be used as the base.)
3. Use your ruler to measure the height of the tetrahedron when it is sitting on a table or your desk. The height of the triangle should be measured by putting the ruler base flat on the table and standing the ruler up straight to see how high the peak of the tetrahedron reaches. **DO NOT** measure along the side of the tetrahedron by laying the ruler on the tetrahedron (this would be the slant height.)
4. Compute the volume of a tetrahedron by using the pyramid volume formula:  
$$V = \frac{1}{3}Bh$$
.  $B$  is the area of the base (the triangle area you computed in 2) and  $h$  the height from the base to the apex.

5. Use dot paper to draw several different shapes of tetrahedrons. These show examples of how the tetrahedron shape changes as the vertices (or satellites) change positions. Instructions: (1) Connect several dots in a line to form the base of a triangle. (2) Choose another dot **not** on the line to be your third vertex of the triangle. (3) Use a ruler to draw the other two sides of the triangle, connecting the third vertex to the line. (4) Now to form a tetrahedron, choose another dot outside of the triangle and connect all three vertices of the triangle to it. Create three different tetrahedra on your dot paper.



## Appendix H: Spacecraft Solar Power Worksheet

Go to the website you or your group has been assigned and find facts about how NASA Missions use solar power to power spacecrafts and vehicles. Answer the following questions if possible, draw a diagram of the solar panels, and make note of other interesting facts.

Name of NASA Mission \_\_\_\_\_

Type of spacecraft or vehicle that uses solar power  
\_\_\_\_\_

Duration of mission \_\_\_\_\_

What is will be powered by the solar panels?  
\_\_\_\_\_

Configuration of solar array panels used (Draw diagram below).

# Appendix I: Worksheet for Computing the Electrical Power Generated by the Solar Panels

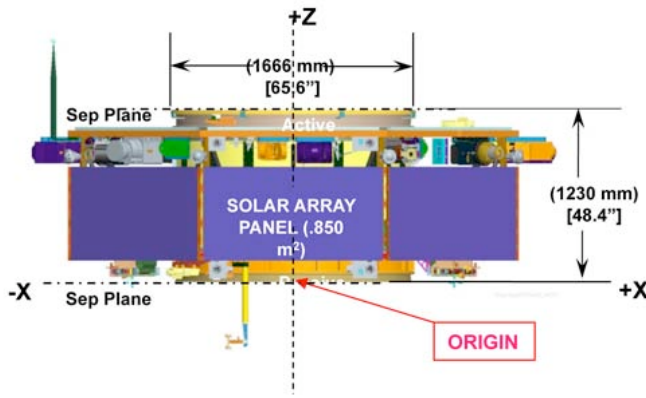


Figure 2: Diagram of MMS Satellite showing the Solar Array Panel

The MMS Satellites utilize solar array panels to produce electricity to power the systems on the satellite. Figure 1 illustrates how the solar arrays are positioned on the satellite and shows the size of the solar array panel is 0.850 meters squared.

- Convert the size of the panel to square centimeters.  
(Recall  $100 \text{ cm} = 1 \text{ m}$  and  $\text{m} \times \text{m} = \text{m}^2$ )  
 $0.085 \text{ m}^2 = \underline{\hspace{2cm}} \text{ cm}^2$

- Suppose the solar array panel can produce 0.03 watts of power **per square centimeter** ( $\text{cm}^2$ ). Then how much power can each panel produce?
- For solar energy to be collected by the panel, it must be facing (or exposed to) the Sun. Use the next page to help you investigate how many solar panels can be facing (or exposed to) the Sun at one time. Note that the number of faces that are exposed to the Sun changes as the satellite rotates. Cut out the octagon that represents the satellite and then place it in the Sun Rays to see how many faces can be directly hit by the rays. Turn the spacecraft slowly in the Sun Rays to see how many sides can become exposed. If the front of the panel cannot “see” the Sun then the panel does not count.
  - What is the minimum number of panels that are exposed to the Sun?
  - What is the maximum number of panels that are exposed to the Sun?

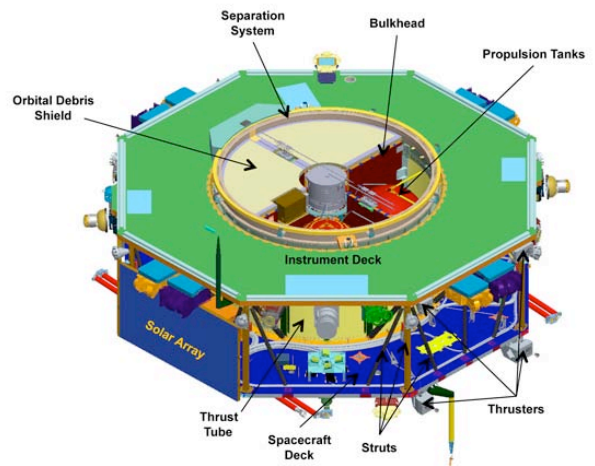
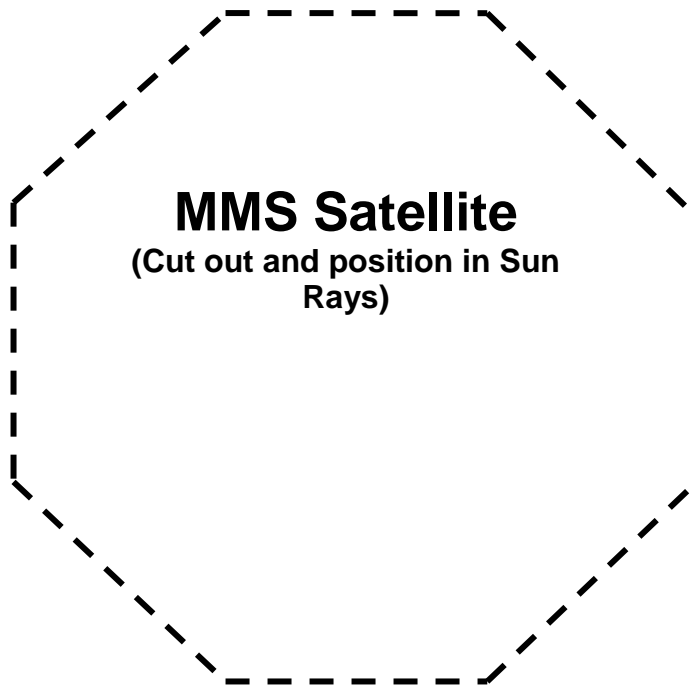
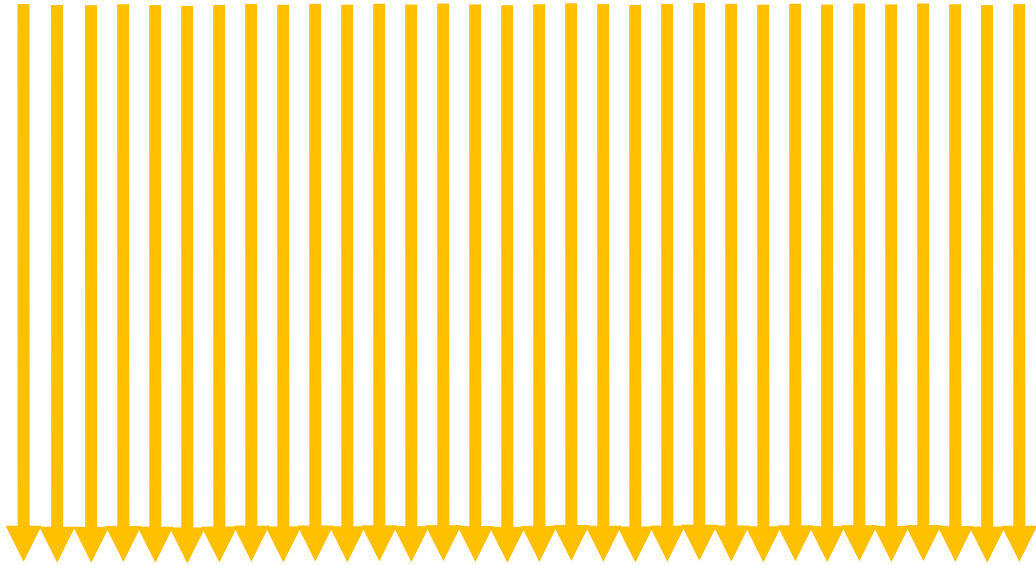


Figure 3: Diagram showing the Octagonal Shape of the Satellite

- How much power is generated when the minimum number of panels is facing the Sun?  
Minimum Power = (don't forget the units on your answer)
- How much power is generated when the maximum number of panels is facing the Sun?  
Maximum Power = (don't forget the units on your answer)

## SUN RAYS



## Appendix J: Answer Keys

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### Appendix C: Worksheet for Computing the Area of the Top and Bottom of MMS Satellite

1. Octagon
2. Method 1 – Divide the octagon into eight triangular parts  
Method 2 – Divide the octagon into a rectangle and two trapezoids
3. Note for this problem that measurements are approximate; the symbol ~ means “approximately”  
Method 1 – Divide the octagon into eight triangular parts each with base of 1.34 meters and height of 1.75 meters. Each triangle has area of  $\frac{1}{2}bh = \frac{1}{2} * 1.45 * 1.75 = 1.26875$  square meters. Multiply this by 8 for the 8 triangles would be  $8 * 1.26875 = 10.15$  square meters.  
  
Method 2 – Divide the octagon into a rectangle and two trapezoids. The rectangle has length 3.5meters and width 1.34 meters.  $L * W = 5.075$  square meters. For the trapezoid, base 1 = 3.5 and base 2 = 1.45. The height is  $(3.5 - 1.45) / 2 = 1.025$ . The area of the trapezoid is  $(h/2)(b1+b2) = (1.025/2)(3.5+1.45) = 2.537$  square meters. Add the areas of all three shapes  $5.075 + 2.537 + 2.537 = 10.15$  square meters.
4. If the apothem (length from the center of the octagon to the middle of a side) is 1.75 meters, then the radius (length from the center of the octagon to one of the vertices) is ~1.8942 meters. The radius of a circle that circumscribes the octagon must have radius greater than 1.8942 meters.

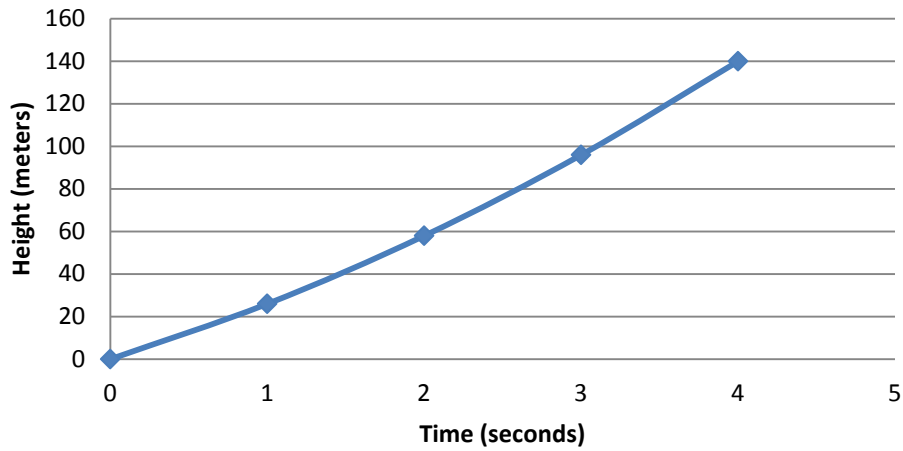
### Appendix D: Rocket Worksheet

1. Satellites
2. a million
3. 6,000
4. 22,000
5. the desired orbit or on its desired trajectory
6. 860,000 lbs.
7. Liquid oxygen/liquid kerosene
8. 4 meters
9. In the upper part of the second stage.
10. Short, medium, and large

Appendix E:

1.

### Time Versus Height

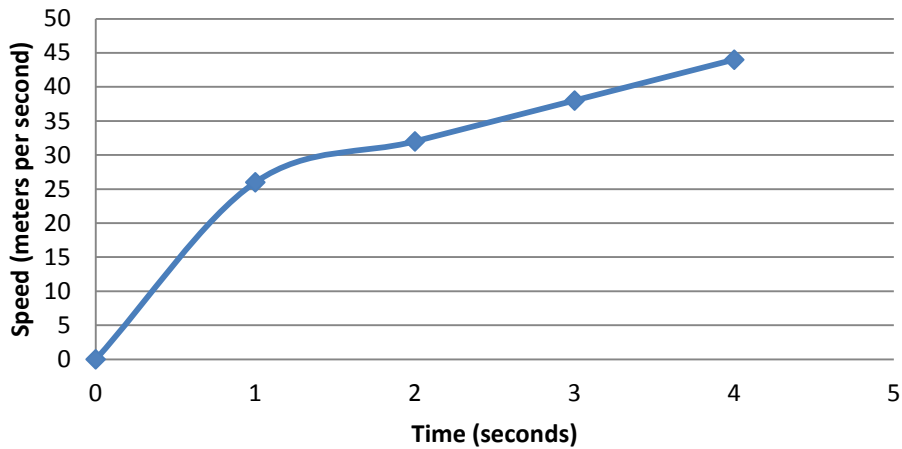


2.

Time (sec)	Height (m)	Speed (m/sec)
0	0	0
1	26	26
2	58	32
3	96	38
4	140	44

3.

### Time versus Speed



4. At approximately 9 to 10 seconds.



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### Appendix F: Formation Flying Worksheet

1. Geese
2. Satellite formation flying is the concept where multiple satellites work together in a group to accomplish an objective.
3. The Earth Observing Satellites (this is one example, but there are others)
4. The EOS collects data about the Earth for researchers and practitioners to use.
5. Landsat 7, CALIPSO, CloudSat, Terra, Aqua

### Appendix G: Directions for Computing the Volume of the Flight Configuration

1. The picture shows the assembled tetrahedron



2. The height of the triangle is 2.5 inches and the base is 3 inches. The area is  $\frac{1}{2} * 2.5 * 3 = 3.75$  square inches.
3. The height of the tetrahedron is approximately 2.5 inches.
4.  $V = \frac{1}{3} * 3.75$  square inches \* 2.5 inches = 3.125 inches cubed.

### Appendix H: Spacecraft Solar Power Worksheet

There will be various answers depending on mission.

### Appendix I: Worksheet for Computing the Electrical Power Generated by the Solar Panels

1. 850
2. 25.5 watts
3. a. 2 panels  
b. 3 panels
4. 51 watts
5. 76.5 watts

## Appendix L: Lesson Summary and Standards Mapping

Lesson	Objectives	NCTM Standards	Common Core Standards
<b>1: Model of the MMS Satellite</b>	<p>Build a three dimensional scale paper model of one of the MMS satellites.</p> <p>Calculate the octagonal area of the top and bottom of the satellite, given the measurements from the satellite.</p> <p>Compare the octagonal cross section area of the satellite with the circular cross section area of the launch vehicle to determine if the space craft will fit the cargo bay.</p>	<p>GEOMETRY: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships</p> <ul style="list-style-type: none"> <li>precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties</li> </ul> <p>ADD SPECIFIC STATE STANDARDS HERE.</p>	<p>GEOMETRY:</p> <p>5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</p> <p>6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</p> <p>7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</p> <p>7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p> <p>7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</p> <p>7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p>
<b>2: Launch of the Satellites</b>	<p>Compute the speed of the launch rocket, given a data chart of time vs. distance data</p>	<p>ALGEBRA: <u>Understand patterns</u>, relations, and functions</p> <ul style="list-style-type: none"> <li>represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic</li> </ul>	<p>5.NF.3. Interpret a fraction as division of the numerator by the denominator (<math>a/b = a \div b</math>). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret <math>3/4</math> as the result of dividing 3 by 4, noting that <math>3/4</math> multiplied by 4 equals 3, and that when 3</p>

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	<p>from lift-off.</p>	<p>rules</p> <ul style="list-style-type: none"> <li>identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations</li> </ul> <p>ALGEBRA: <u>Analyze change</u> in various contexts</p> <ul style="list-style-type: none"> <li>use graphs to analyze the nature of changes in quantities in linear relationships.</li> </ul> <p>ADD SPECIFIC STATE STANDARDS HERE.</p>	<p>wholes are shared equally among 4 people each person has a share of size <math>\frac{3}{4}</math>. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</p> <p>6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p> <p>6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation <math>d = 65t</math> to represent the relationship between distance and time.</p> <p>7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks <math>\frac{1}{2}</math> mile in each <math>\frac{1}{4}</math> hour, compute the unit rate as the complex fraction <math>\frac{1/2}{1/4}</math> miles per hour, equivalently 2 miles per hour.</p> <p>8.F.3. Interpret the equation <math>y = mx + b</math> as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function <math>A = s^2</math> giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.</p> <p>8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p>
<p><b>3: The Satellites Flight Configuration</b></p>	<p>Visualize the three dimensional tetrahedral flight configuration</p>	<p>GEOMETRY: Use visualization, spatial reasoning, and geometric modeling to solve problems</p> <ul style="list-style-type: none"> <li>draw geometric objects with</li> </ul>	<p>5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</p> <p>6.G.2. Find the volume of a right rectangular</p>

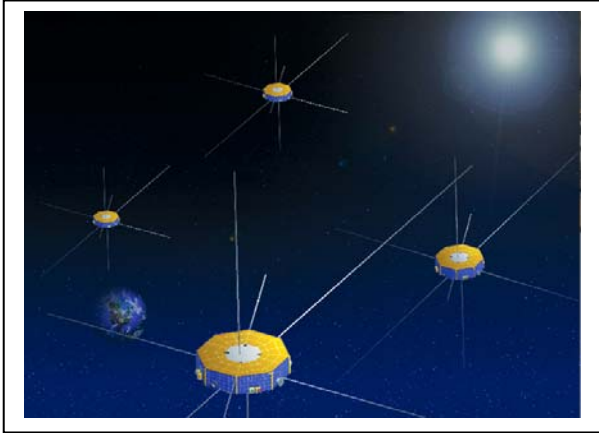
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	<p>of the four satellites in the mission using graphing techniques and models.</p> <p>Analyze, by graphing techniques, the changing shape of the tetrahedron as the satellites change position.</p> <p>Compute the volume of the tetrahedron based on the positions of the four satellites.</p>	<p>specified properties, such as side lengths or angle measures</p> <ul style="list-style-type: none"> <li>• use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;</li> </ul> <p>ADD SPECIFIC STATE STANDARDS HERE.</p>	<p>prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas <math>V = l w h</math> and <math>V = b h</math> to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p> <p>6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</p> <p>7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</p> <p>7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p> <p>7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p> <p>8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</p>
<p><b>4: Powering the Satellites</b></p>	<p>Calculate the surface area of the panels that are exposed to the sun for various positions of the satellite, given the dimensions of the solar panels.</p> <p>Calculate the power generated by the solar panels for various</p>	<p>GEOMETRY: Use visualization, spatial reasoning, and geometric modeling to solve problems</p> <ul style="list-style-type: none"> <li>• use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume</li> <li>• use geometric models to represent and explain numerical and algebraic relationships</li> <li>• recognize and apply geometric ideas and relationships in</li> </ul>	<p>8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</p>

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	<p>positions of the satellite, given the dimensions of the panels.</p> <p>Organize and write a report about the satellites in the MMS Mission that contains information from the four lessons, after completing the exercises for the unit.</p>	<p>areas outside the mathematics classroom, such as art, science, and everyday life</p> <p>ADD SPECIFIC STATE STANDARDS HERE.</p>	
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## **SpaceMath@NASA Supplemental Problems**



In 2014, the four satellites of the Magnetosphere Multiscale (MMS) mission will be launched into orbit atop an Atlas-V421 rocket. These satellites, working together, will attempt to measure dynamic changes in Earth's magnetic field that physicists call magnetic reconnection events. These changes in the magnetic field are responsible for many different phenomena including Earth's polar aurora.

Soon after launch, the satellites will be placed into a Phase-1 elliptical orbit with Earth at one of the foci. The closest distance between the satellite and Earth, called perigee, will be at a distance of  $1.2 R_e$ , where  $1.0 R_e$  equals the radius of Earth of 6,378 km. The farthest distance from Earth, called apogee, occurs at a distance of  $12.0 R_e$ . After a few years, the satellites will be moved into a Phase-2 elliptical orbit with an apogee of  $25.0 R_e$  and a perigee of  $1.2 R_e$ .

The standard form for an ellipse is 
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where  $x$  and  $y$  are the coordinates of a point on the ellipse, and the ellipse constants  $a$  and  $b$ , which are the semi-major and semi-minor axis dimensions of the ellipse. The relationship between the apogee ( $A$ ) and perigee ( $P$ ) distances, and the ellipse constants,  $a$  and  $b$ , and the eccentricity of the ellipse,  $e$ , are as follows:

$$A = a + c \quad P = a - c \quad e = c/a \quad b = a(1 - e^2)^{1/2}$$

**Problem 1** – What are the equations for the semi-major and semi-minor axis,  $a$  and  $b$ , in terms of only  $A$  and  $P$ ?

**Problem 2** – What are the equations of the MMS elliptical orbit in standard form, with all distances given in multiples of  $R_e$  for A) Phase-1 and B) Phase-2?

**Problem 3** – What are the equations of the MMS elliptical orbit in the form  $kx^2 + gy^2 = s$  where  $k$ ,  $g$  and  $s$  are numerical constants rounded to integers for A) Phase-1 and B) Phase-2?

# Answer Key

The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \quad e = c/a \quad b = a(1 - e^2)^{1/2}$$

**Problem 1** – What are the equations for the semi-major and semi-minor axis, a and b, in terms of only A and P?

Answer: Add the equations for A and P to get  $A + P = 2a$ , and  $a = (A + P)/2$   
Subtract the equations for A and P to get  $A - P = 2c$ , and  $c = (A - P)/2$   
Then by substitution  $e = (A - P)/(A + P)$

$$\text{And so } b = \frac{(A + P)}{2} \sqrt{1 - \frac{(A - P)^2}{(A + P)^2}} \quad \text{so } b = \frac{1}{2} \sqrt{(A + P)^2 - (A - P)^2}$$

Expand and simplify:

$$b = \frac{1}{2} (A^2 + 2AP + P^2 - A^2 + 2AP - P^2)^{1/2}$$
$$b = \frac{1}{2} (4AP)^{1/2}$$
$$b = (AP)^{1/2}$$

**Problem 2** –

Answer: A) for  $A = 12.0$  and  $P = 1.2$ ,  $a = (13.2/2) = 6.6 R_e$   $c = (12 - 1.2)/2 = 5.4 R_e$   
 $e = (5.4/6.6) = 0.82$   
 $b = (12 \times 1.2)^{1/2} = 3.8 R_e$

$$\text{then } 1 = \frac{x^2}{(6.6)^2} + \frac{y^2}{(3.8)^2}$$

B) for  $A = 25.0$  and  $P = 1.2$ ,  $a = (26.2/2) = 13.1 R_e$   $c = (25 - 1.2)/2 = 11.9 R_e$   
 $e = (11.9/13.1) = 0.91$   
 $b = (25 \times 1.2)^{1/2} = 5.5 R_e$

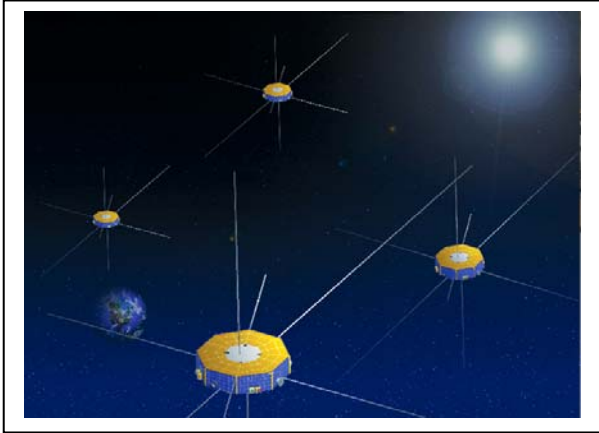
$$\text{then } 1 = \frac{x^2}{(13.1)^2} + \frac{y^2}{(5.5)^2}$$

**Problem 3** –

Answer: A)  $1 = x^2/(6.6)^2 + y^2/(3.8)^2$  cross-multiply and simplify to get  
 $(6.6)^2(3.8)^2 = (3.8)^2x^2 + (6.6)^2y^2$   
 $662.55 = 14.4x^2 + 43.56y^2$  and rounded to the nearest integer:  
 $663 = 14x^2 + 44y^2$

B)  $1 = x^2/(13.1)^2 + y^2/(5.5)^2$  cross-multiply and simplify to get  
 $(13.1)^2(5.5)^2 = (5.5)^2x^2 + (13.1)^2y^2$   
 $5191.20 = 30.25x^2 + 171.61y^2$  and rounded to the nearest integer:  
 $5191 = 30x^2 + 172y^2$





In 2014, the four satellites of the Magnetosphere Multiscale (MMS) mission will be launched into orbit atop an Atlas V421 rocket. These satellites, working together, will attempt to measure dynamic changes in Earth's magnetic field that physicists call magnetic reconnection events. These changes in the magnetic field are responsible for many different phenomena including Earth's polar aurora.

Soon after launch, the satellites will be placed into a Phase-1 elliptical orbit with Earth at one of the foci. After a few years, the satellites will be moved into a different 'Phase-2' elliptical orbit. The closest distance between the satellite and Earth is called the perigee. The farthest distance from Earth is called the apogee. The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \quad e = c/a \quad b = a(1 - e^2)^{1/2}$$

Where  $a$ ,  $b$  and  $e$  are the semi-major and semi-minor axis lengths, and  $e$  is the eccentricity of the ellipse. The equations describing the Phase-1 and Phase-2 orbits are as shown below, with all distance units given in terms of multiples of 1 Earth radius ( $1 R_E = 6,378 \text{ km}$ ):

$$\text{Phase-1} \quad 663 = 14x^2 + 44y^2$$

$$\text{Phase-2} \quad 5191 = 30x^2 + 172y^2$$

**Problem 1** – In terms of kilometers, and to two significant figures, what is the semi-major axis distance,  $a$ , for the A) Phase-1 orbit? B) Phase-2 orbit?

**Problem 2** – According to Kepler's Third Law, for orbits near Earth, the relationship between the semi-major axis distance,  $a$ , and the orbital period,  $T$ , is given by

$$a^3 = 287 T^2$$

where  $a$  is in units of  $R_E$  and  $T$  is in days. To the nearest tenth of a day, what are the estimated orbit periods for the MMS satellites in A) Phase-1? B) Phase-2?

# Answer Key

The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \quad e = c/a \quad b = a(1-e^2)^{1/2}$$

The equations describing the Phase-1 and Phase-2 orbits are as shown below, with all distance units given in terms of multiples of 1 Earth radius ( $1 R_e = 6,378 \text{ km}$ ):

Phase-1  $663 = 14x^2 + 44y^2$

Phase-2  $5191 = 30x^2 + 172y^2$

**Problem 1** – In terms of kilometers, and to two significant figures, what is the semi-major axis distance,  $a$ , for the A) Phase-1 orbit? B) Phase-2 orbit?

Answer: A) Writing the Phase-1 equation in standard form:

$$1 = \frac{x^2}{(6.9)^2} + \frac{y^2}{(3.9)^2}$$

then  $a = 6.9 R_e$   
 $= 6.9 \times (6,378 \text{ km})$   
 $= 44,008 \text{ km.}$   
 $= 44,000 \text{ km, to 2 SF}$

B) Writing the Phase-2 equation in standard form:

$$1 = \frac{x^2}{(13.1)^2} + \frac{y^2}{(5.5)^2}$$

then  $b = 13.1 R_e$   
 $= 13.1 \times (6,378 \text{ km})$   
 $= 83,552 \text{ km.}$   
 $= 84,000 \text{ km, to 2 SF}$

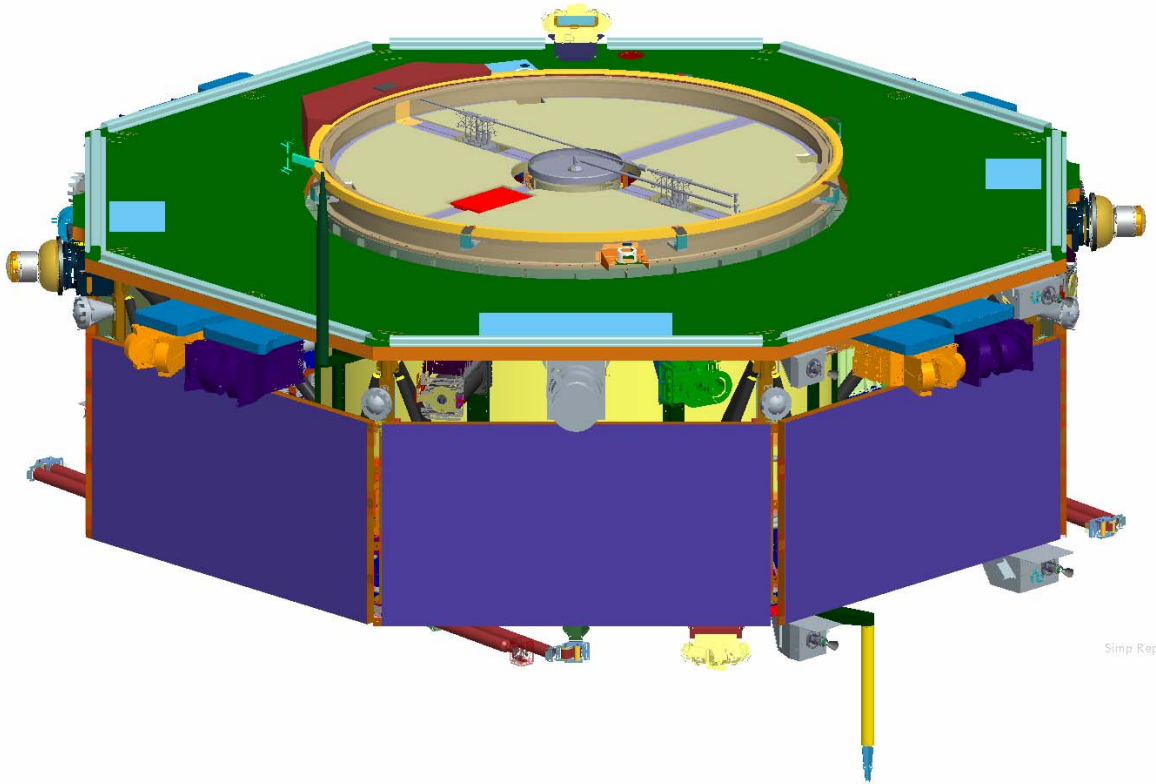
**Problem 2** – According to Kepler's Third Law, for orbits near Earth, the relationship between the semi-major axis distance,  $a$ , and the orbital period,  $T$ , is given by

$$a^3 = 287 T^2$$

where  $a$  is in units of  $R_e$  and  $T$  is in days. To the nearest tenth of a day, what are the estimated orbit periods for the MMS satellites in A) Phase-1? B) Phase-2?

Answer: A)  $(6.9)^3 = 287 T^2$ , so for Phase-1,  $T = 1.1$  days.

B)  $(13.1)^3 = 287 T^2$ , so for Phase-2,  $T = 2.8$  days.



Each of the Magnetosphere Multiscale (MMS) satellites is in the shape of an octagonal prism. The faces of the satellite are partially covered with solar cells that will generate the electricity to operate the satellite and its many experiment modules.

**Problem 1** - If the distance between opposite faces of the satellite is  $D = 3.150$  meters, and the height of each solar panel is  $h = 0.680$  meters, what is the width of one rectangular solar panel if  $w = 0.414 D$ ?

**Problem 2** - What is the surface area of one rectangular solar panel in the satellite?

**Problem 3** If the solar cells generate 0.03 watts per square centimeter of surface area, how much power will be generated by a single face of the satellite?

## Answer Key

**Problem 1** - If the distance between opposite faces of the satellite is  $D = 3.150$  meters, and the height of each solar panel is  $h = 0.680$  meters, what is the width of one rectangular solar panel if  $w = 0.414 D$ ?

Answer: The problem says that  $D = 3.150$  meters, so  $w = 0.414 (3.150) = \mathbf{1.30}$  meters.

Advanced students who know trigonometry can determine the width from the following geometry:

The relevant right-triangle has two sides that measure  $D/2$  and  $w/2$  with an angle of  $45/2 = 22.5$ . Then from trigonometry,  $\tan(45/2) = w/D$ . Since  $\tan(22.5) = 0.414$ , we have  $w = 0.414 (D)$ , and so  $\mathbf{w = 1.30}$  meters.

**Problem 2** - The area of each solar panel is just  $A = (1.30 \text{ meters}) \times (0.680 \text{ meters})$  so

$$\text{Area} = \mathbf{0.88 \text{ meter}^2}.$$

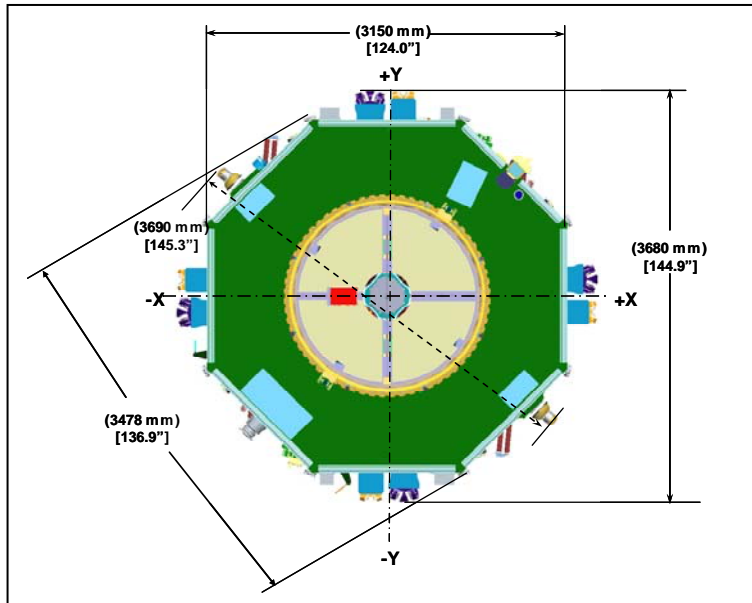
**Problem 3** - If the solar cells generate 0.03 watts per square centimeter of surface area, how much power will be generated by a single face of the satellite?

Answer: A single face of the satellite has a solar panel with an area of  $0.88 \text{ meters}^2$ . First convert this into square centimeters

$$\begin{aligned} A &= 0.88 \text{ meter}^2 \times (100\text{cm} / 1 \text{ m}) \times (100 \text{ cm}/1 \text{ m}) \\ &= 8800 \text{ cm}^2 \end{aligned}$$

Then multiply by the solar power area factor of  $0.03 \text{ watts/cm}^2$  to get

$$\text{Power} = \mathbf{264 \text{ watts}}.$$



The Magnetosphere Multiscale (MMS) constellation consists of 4 identical satellites that will be launched into orbit in 2014 to investigate Earth's magnetic field in space. Each satellite is an octagonal prism with a face-to-face diameter of 3.15 meters and a height of 0.896 meters. A central, cylindrical hole has been cut out of each satellite to accommodate the steering rockets and fuel tanks. This cylindrical hole has a diameter of 1.66 meters.

**Problem 1** - What is the formula for the area of an octagon with a diameter of  $D$  meters, if the area of one of the 16 inscribed right triangles is given by the formula  $A = 0.104D^2$ ?

**Problem 2** - To three significant figures, what is the total surface area, including side faces, of a single MMS satellite? (Hint: don't forget the cylindrical hole!)

**Problem 1** - What is the formula for the surface area of an octagon with a diameter of D meters if the area of one of the 16 inscribed right triangles is given by the formula  $A = 0.104D^2$ ?

Answer: Note: Draw an octagon with the stated dimensions. Reduce the octagonal area to the areas of 16 right-triangles with sides w and D/2. Using trigonometry,  $w/2 = D/2 \tan(45/2)$  so  $w = 0.414 D$ , and then the area of a single triangle is  $A = 1/2 (D/2) \times (0.414D)$  or  $A = 0.104 D^2$ .

The total area of a single octagonal face is then  $A = 16 \times (0.104) D^2$ , or  **$A = 1.664D^2$** .

**Problem 2** - To three significant figures, what is the total surface area, including side faces, of a single MMS satellite?

Answer: The surfaces consist of 2 octagons, and 8 rectangular side panels. The total area is then  $A = 2 (1.664D^2) + 8 (h)(w)$ . For  $D = 3.15$  meters,  $h = 0.896$  meters and  $w = 0.414(3.15) = 1.30$  meters, we have a total area of

$A = 2(1.664)(3.15)^2 + 8(0.896)(1.30) = 42.34 \text{ meters}^2$ . This is for a solid, regular octagonal cylinder. However, for an MMS satellite, a cylindrical hole has been removed. This means that for the top and bottom faces, a circular area of  $A = 2 \times \pi (1.66/2)^2 = 4.33 \text{ meters}^2$  has been removed, and a surface area for the inside cylindrical hole has been ADDED. This surface area is just  $A = 2 \pi r h$  or  $A = 2 (3.14) (1.66/2)(0.896) = 4.67 \text{ meters}^2$ . So the total area is just

$$A = 42.34 \text{ meters}^2 - 4.33 \text{ meters}^2 + 4.67 \text{ meters}^2$$

$$A = 42.68 \text{ meters}^2, \text{ which to three SF is just } \mathbf{A = 42.7 \text{ meters}^2}.$$

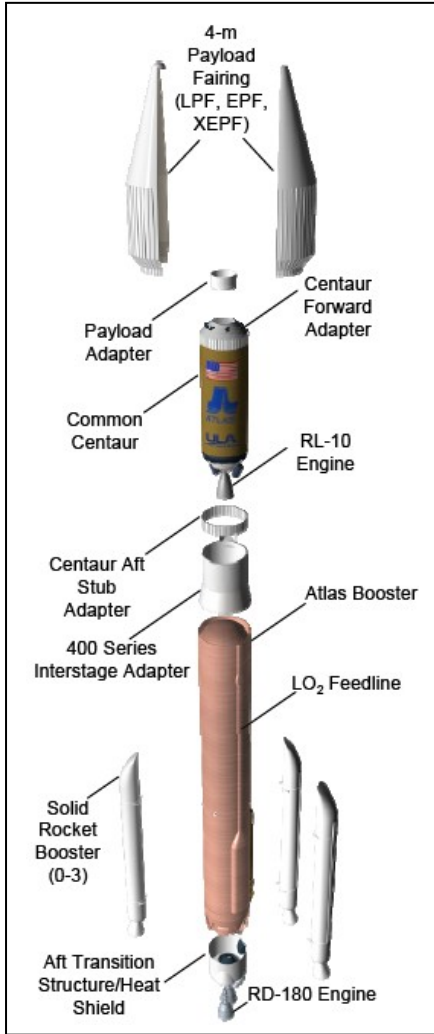
**Problem 3** - To three significant figures, what is the volume of a single MMS satellite?

Answer: The surface area of a single satellite hexagonal face is  $A = 1.664D^2$  and the volume is just  $V = 1.664D^2 h$ . With a central cylindrical volume subtracted with  $V_c = \pi r^2 h$ , then  $V = h (1.664D^2 - \pi r^2)$ . For a single MMS satellite

$$V = (0.896)(1.664(3.15)^2 - 3.141(1.66/2)^2)$$

$$V = 14.79 - 1.94$$

$$V = 12.85 \text{ meters}^3. \text{ so to three SF we have } \mathbf{V = 12.9 \text{ meters}^3}.$$



The Magnetosphere Multiscale (MMS) satellite constellation will be launched into orbit in 2014 atop an Atlas V421XEPF rocket. There are dozens of different configurations of Atlas rockets depending on the mass and orbit destination of the payload being launched. One of these that is similar to the MMS rocket is shown in the figure to the left. The Atlas V421 consists of single Atlas rocket booster with two strap-on solid rocket boosters, and a second-stage Centaur rocket booster with the MMS satellite stack attached above the Centaur.

Atlas: Height = 32.46 meters  
 Diameter = 3.81 meters  
 Mass = 399,700 kg

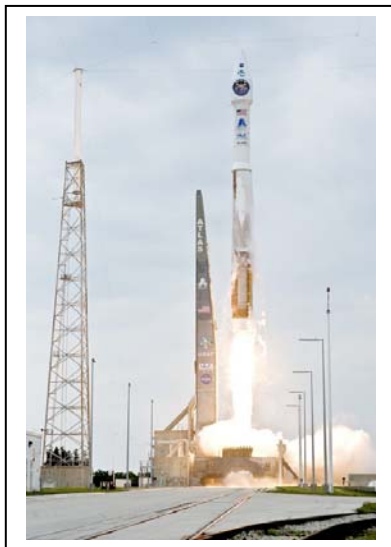
Centaur: Height = 12.68 meters  
 Diameter = 3.05 meters  
 Mass = 23,100 kg

Payload: Height = 13.81 meters  
 Diameter = 4.2 meters  
 Mass = 7,487 kg

**Problem 1** - What is the total length of the MMS launch vehicle in A) meters? B) feet? C) inches? (Note: 1 meter = 3.281 feet)

**Problem 2** - To the nearest tenth of a percent, what percentage of the launch vehicle height is occupied by the payload?

**Problem 3** - To the nearest tenth of a percent, if each satellite has a mass of 1,250 kg, what percentage of the entire rocket mass is occupied by the four satellites?



## Answer Key

Atlas: Height = 32.46 meters  
Diameter = 3.81 meters      Mass = 399,700 kg

Centaur: Height = 12.68 meters  
Diameter = 3.05 meters      Mass = 23,100 kg

Payload: Height = 13.81 meters  
Diameter = 4.2 meters      Mass = 7,487 kg

**Problem 1** - What is the total length of the MMS launch vehicle in A) meters? B) feet? C) Inches?

Answer: A)  $H = 32.46 \text{ meters} + 12.68 \text{ meters} + 13.81 \text{ meters} = \mathbf{58.95 \text{ meters}}$ .  
B)  $H = 58.95 \text{ meters} \times (3.281 \text{ feet/meters}) = \mathbf{193.41 \text{ feet}}$ .  
C)  $H = 193.41 \text{ feet} \times (12 \text{ inches} / 1 \text{ foot}) = \mathbf{2320.92 \text{ inches}}$ .

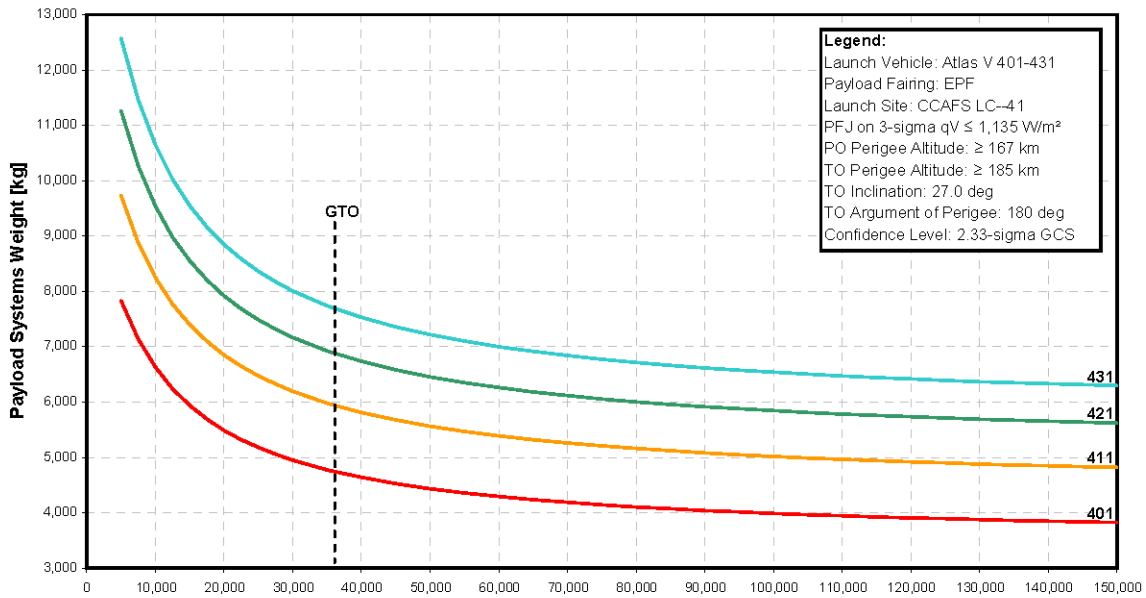
**Problem 2** - To the nearest tenth of a percent, what percentage of the launch vehicle height is occupied by the payload?

Answer: Total length = 58.95 meters. Payload = 13.81 meters so  
 $P = 100\% (13.81/58.95) = \mathbf{23.4\%}$

**Problem 3** - To the nearest tenth of a percent, if each satellite has a mass of 1,250 kg, what percentage of the entire rocket mass is occupied by the four satellites?

Answer: The combined masses for the Atlas booster, the Centaur upper stage and the payload is  $M = 399,700 \text{ kg} + 23,100 \text{ kg} + 2,487 \text{ kg} + 4(1,250 \text{ kg}) = 430,287 \text{ kg}$ . The mass in the satellites is  $m = 4(1,250 \text{ kg}) = 5000 \text{ kg}$ , so the percentage is just  
 $P = 100\% (5000/430287) = \mathbf{1.2\%}$ .





This figure, obtained from the Boeing Corporation ‘Atlas V Users Guide: 2010’ shows the maximum payload mass that can be launched into a range of orbits with apogees from 6,378 km to 150,000 km. The apogee distance is the maximum distance from the center of Earth that the orbit will take the payload. The specific models of Atlas rocket are given on the right-hand edge and include the Atlas V401, V411, V421 and V431. For example, the Atlas V401 can lift any payload with a mass of less than 4,000 kg into an orbit with an apogee of no more than 100,000 km.

**Problem 1** – For a particular Atlas V model, what does the curve for that model tell you about payload mass and maximum altitude?

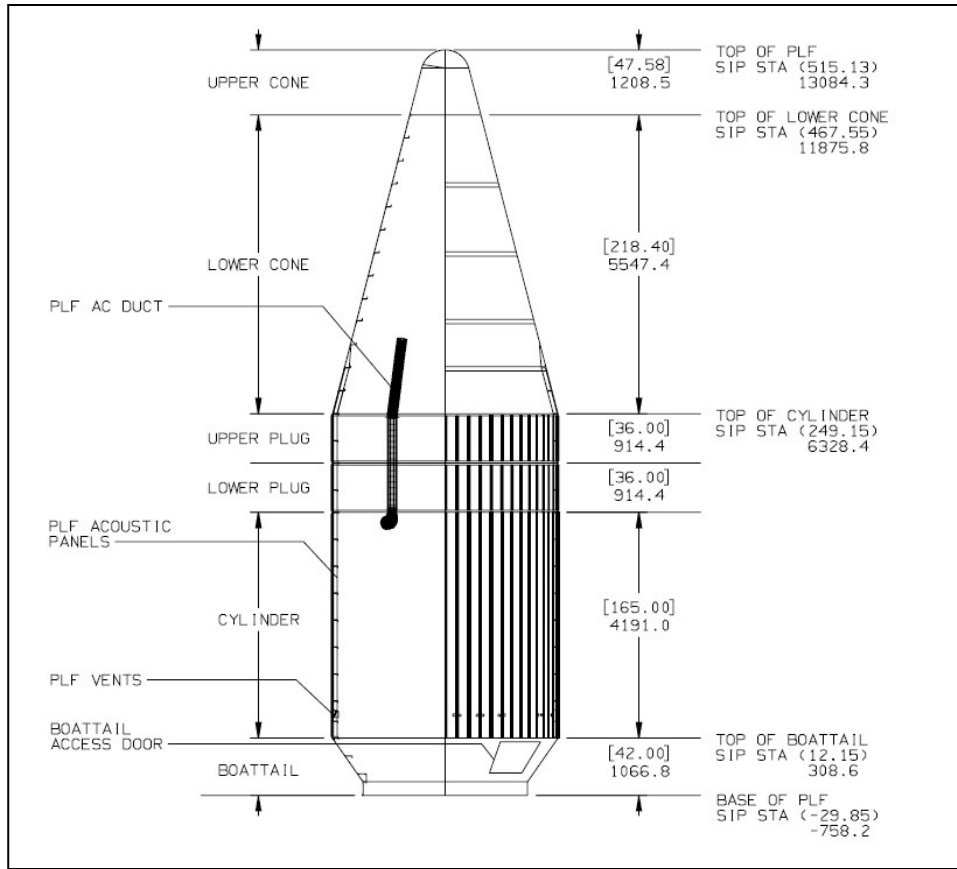
**Problem 2** – Suppose a scientist wanted to place a 6,500 kg research satellite into orbit with an apogee of 70,000 km. What is the best choice of launch vehicle to satisfy this requirement?

**Problem 1** – For a particular Atlas V model, what does the curve for that model tell you about payload mass and maximum altitude?

Answer: The curves all decline in payload mass as the orbit apogee increases. This means that there is a trade-off between the mass you can place into orbit and the maximum apogee for that mass. The less mass your payload has, the greater is the apogee of the orbit that you can reach. This is the same concept as it is easier for you to throw a 3 ounce tennis ball to a higher altitude than a 2-pound lead weight! The same relationship works for rockets.

**Problem 2** – Suppose a scientist wanted to place a 6,500 kg research satellite into orbit with an apogee of 70,000 km. What is the best choice of launch vehicle to satisfy this requirement?

Answer: We find '70,000 km' on the horizontal axis and draw a vertical line. Next we draw a horizontal line from '6,500 kg' on the vertical axis until it intercepts our vertical line at '70,000 km'. The intersection point lies just above the curve for the V 421, meaning that it is not powerful enough for the task, but it lies below the curve for the **Atlas V431** launch vehicle, so this vehicle has the capacity to launch this payload to the indicated apogee.



The four satellites of the Magnetosphere Multiscale (MMS) mission will be stacked vertically inside a nose-cone payload shroud, which will be jettisoned when the payload reaches orbit. The diagram above shows the dimensions of the payload. The bracketed numbers are in inches. The numbers below the brackets are the corresponding dimensions in millimeters. The diameter of the cylindrical section is 4.2 meters. The cylindrical payload section is topped by a conical ‘nose-cone’ section, which in turn comes to an end in a hemispherical cap with a diameter of 910 millimeters.

**Problem 1** – To 2 significant figures, what is the volume of the cylindrical payload section including the Upper and Lower Plugs; A) in cubic meters? B) in cubic centimeters? C) in cubic feet? (1 foot = 30.48 cm).

**Problem 2** – To 2 significant figures, what is the volume for the hemispherical cap at the top of the nose-cone in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

**Problem 3** – The formula for the volume of a truncated right circular cone is given by  $V = \frac{1}{3} \pi (R^2 + rR + r^2)h$ , where  $R$  is the radius at the base, and  $r$  is the radius at the truncation. To 2 significant figures, what is the volume of the conical section in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

**Problem 1** – To 2 significant figures, what is the volume of the cylindrical payload section including the Upper and Lower Plugs A) in cubic meters? B) in cubic centimeters? C) in cubic feet? (1 foot = 30.48 cm)

Answer: A)  $r = 4.2/2 = 2.1$  meters, and converting the measurements in millimeters to meters,  $h = 4.191\text{m} + 0.914\text{m} + 0.914\text{m} = 6.019\text{m}$ ,  
so  $V = \pi r^2 h = 3.141 (2.100)^2 (6.019) = \mathbf{83 \text{ meters}^3}$ .

B)  $V = 83.0 \text{ m}^3 \times (100 \text{ cm} / 1\text{m})^3 = \mathbf{83,000,000 \text{ cm}^3}$ .

C)  $V = 83000000 \text{ cm}^3 \times (1 \text{ foot} / 30.48 \text{ cm})^3 = \mathbf{2,900 \text{ feet}^3}$ .

**Problem 2** – To 2 significant figures, what is the volume for the hemispherical cap at the top of the nose-cone in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

Answer: A)  $V = 2/3 \pi R^3$  and  $R = 0.91 \text{ m}/2 = 0.455 \text{ m}$ , so  $V = \mathbf{0.20 \text{ meters}^3}$ .

B)  $V = 0.20 \text{ m}^3 \times (100 \text{ cm}/1 \text{ m})^3 = \mathbf{200,000 \text{ cm}^3}$

C)  $V = 200,000 \text{ cm}^3 \times (1 \text{ foot}/30.48 \text{ cm})^3 = \mathbf{7.0 \text{ feet}^3}$

**Problem 3** – The formula for the volume of a truncated circular cone is given by  $V = 1/3 \pi (R^2 + rR + r^2)$ , where R is the radius at the base, and r is the radius at the truncation. To 2 significant figures, what is the volume of the conical section in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

Answer: A)  $R = 4.2 \text{ m}/2 = 2.1$  meters, and  $r = 0.910 \text{ m}/2 = 0.455 \text{ m}$ , so  
 $V = 0.333 (3.141) (2.1^2 + (2.1)(0.455) + 0.455^2) = \mathbf{5.8 \text{ meters}^3}$ .

B)  $V = 5.8 \text{ m}^3 \times (100 \text{ cm} / 1 \text{ m})^3 = \mathbf{5,800,000 \text{ cm}^3}$ .

C)  $V = 5,800,000 \text{ cm}^3 \times (1 \text{ foot} / 30.48 \text{ cm})^3 = \mathbf{210 \text{ feet}^3}$ .



The Magnetosphere Multiscale (MMS) mission will be launched from Pad 41 at the Cape Canaveral US Air Force Launch Facility in Florida in 2014. The image above shows a spectacular satellite view of this complex. The short, horizontal line to the lower left indicates a length of 100 meters.

**Problem 1** – With the help of a millimeter ruler, what is the scale of this image in meters per millimeter?

**Problem 2** – The circular road is centered on the location of the launch pad for the Atlas V rocket. What is the circumference of this road to the nearest meter?

**Problem 3** – At a comfortable walking pace of 1.5 meter/sec, to the nearest minute, how long would it take you to walk around this perimeter road?

**Problem 4** – There are four towers surrounding the launch pad, and their shadows can be seen pointing to the upper left. If a tower is 73 meters tall, create a scale model showing the horizontal shadow length and the vertical tower height, and determine the angle of the sun at the time the image was made.

## Answer Key

**Problem 1** – With the help of a millimeter ruler, what is the scale of this image in meters per millimeter?

Answer: For ordinary reproduction scales, students should measure the '100 meter' bar to be 12 mm long, so the scale of the image is  $100 \text{ meters}/12 \text{ mm} = \mathbf{8.3 \text{ meters/mm}}$ .

**Problem 2** – The circular road is centered on the location of the launch gantry for the Atlas-V rocket. What is the circumference of this road to the nearest meter?

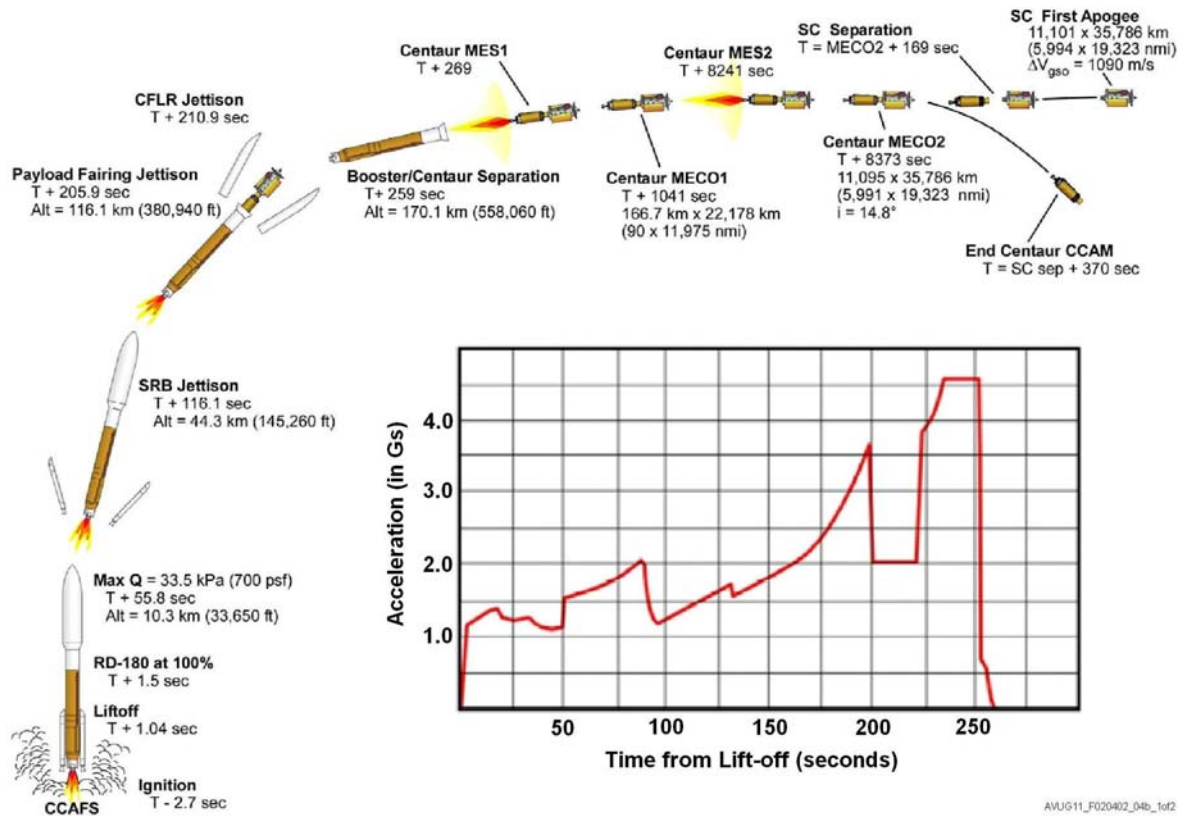
Answer: The diameter of the road is 41 mm or 340 meters. The circumference is  $C = 3.141 \times 340 \text{ meters} = \mathbf{1068 \text{ meters}}$ .

**Problem 3** – At a comfortable walking pace of 1.5 meter/sec, to the nearest minute, how long would it take you to walk around this perimeter road?

Answer:  $T = \text{distance}/\text{speed}$  so  
 $T = 1068 \text{ meters}/1.5 = 712 \text{ seconds}$  or **12 minutes**.

**Problem 4** – There are four towers surrounding the launch gantry, and their shadows can be seen pointing to the upper left. If a tower is 73 meters tall, create a scale model showing the horizontal shadow length and the vertical tower height, and determine the angle of the sun at the time the image was made.

Answer: The shadow of one of the towers measures about 17 mm or 141 meters. The two sides of the right-triangle are therefore 73 meters and 141 meters. Students may draw a scaled model of this triangle, then use a protractor to measure the sun elevation angle opposite the '73 meter' segment. Or they may use  $\tan(\theta) = 73 \text{ meters}/141 \text{ meters}$  and so  $\theta = \mathbf{27 \text{ degrees}}$ .



This diagram, provided by the Boeing *Atlas V Launch Services User's Guide* shows the major events during the launch of an Atlas V521 rocket, which is similar to the rocket planned for the MMS launch in 2014. The figure shows the events in the V521 timeline starting from -2.7 seconds before launch through the Centaur rocket main engine cut off 'MECO2' event at 8,373 seconds after launch. The graph shows the acceleration of the payload during the first 250 seconds after launch. Acceleration is given in Earth gravities, where 1.0 G = 9.8 meters/sec<sup>2</sup>.

- Problem 1** – To the nearest tenth of a G, what is the acceleration at the time of:
- Maximum aerodynamic pressure, called Max-Q ?
  - Solid rocket booster (SRB) jettison?
  - Payload fairing jettison?
  - Booster/Centaur separation?

**Advanced Math Challenge:** The speed of the rocket at a particular time, T, is the area under the acceleration curve (in meters/sec<sup>2</sup>) from the time of launch to the time, T. By approximating the areas as combinations of rectangles and triangles, and rounding your final answers to two significant figures, about what is the rocket speed at a time of:

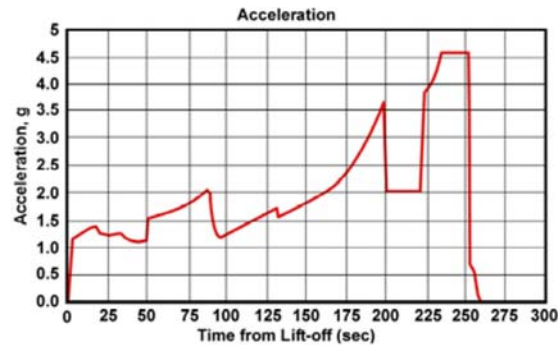
- T = 50 seconds?
- T = 100 seconds?
- T = 200 seconds?
- T = 250 seconds?

# Answer Key

**Problem 1** – To the nearest tenth of a G, what is the acceleration at the time of:

Answer:

- |  |                                       |
|--|---------------------------------------|
| A) Maximum aerodynamic pressure, called Max-Q? | A = 1.6 Gs                            |
| B) Solid Rocket Booster (SRB) jettison?        | A = 1.5 Gs                            |
| C) Payload Fairing Jettison?                   | A = 2.0 Gs                            |
| D) Booster/Centaur Separation?                 | A = 0.0 Gs (thrust no longer applied) |



**Advanced Math Challenge** – The speed of the rocket at a particular time, T, is the area under the acceleration curve (in meters/sec<sup>2</sup>) from the time of launch to the time, T. By approximating the areas as combinations of rectangles and triangles, and rounding your final answers to two significant figures, about what is the rocket speed at a time of:

- A) T = 50 seconds?

$$V = (50 \text{ seconds})(1.3Gs)(9.8 \text{ m/s}^2) = \mathbf{640 \text{ meters/sec.}}$$

- B) T = 100 seconds?

$$\begin{aligned} V &= 640 \text{ meters/sec} + (50 \text{ sec})(1.5Gs)(9.8\text{m/s}^2) + 1/2(50\text{sec})(2.0-1.5)(9.8\text{m/s}^2) \\ &= 640 \text{ meters/sec} + 735 \text{ m/sec} + 123 \text{ m/sec} \\ &= \mathbf{1500 \text{ meters/sec}} \end{aligned}$$

- C) T = 200 seconds?

$$\begin{aligned} V &= 1500 \text{ meters/sec} + (200-100)(1.2 \text{ Gs})(9.8 \text{ m/s}^2) + \frac{1}{2} (200-100)(3.6-1.2)(9.8\text{m/s}^2) \\ &= 1500 \text{ m/sec} + 1200 \text{ m/sec} + 1200 \text{ m/sec} \\ &= \mathbf{3900 \text{ meters/sec.}} \end{aligned}$$

- D) T = 250 seconds?

$$\begin{aligned} V &= 3900 \text{ meters/sec} + (225-200)(2.0Gs)(9.8 \text{ m/s}^2) + (250-225)(4.5 \text{ Gs})(9.8 \text{ m/s}^2) \\ &= 3900 \text{ m/sec} + 490 \text{ m/s} + 1100 \text{ m/s} \\ &= \mathbf{5500 \text{ meters/sec.}} \end{aligned}$$

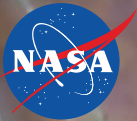
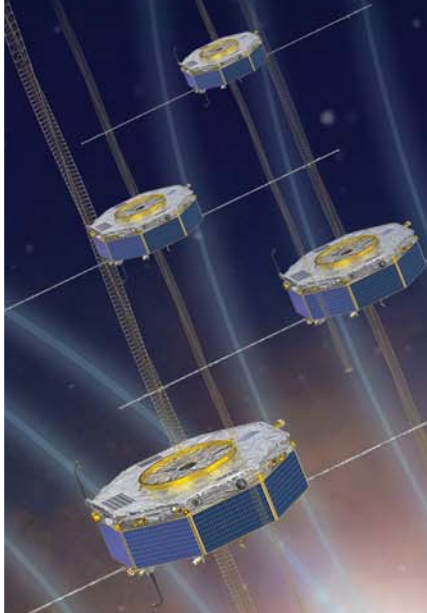
Note: Students answers will vary. The biggest challenge is to convert Gs to meters/sec<sup>2</sup> to get physical units in terms of meters and time.

To reach 'orbit', the payload needs a speed of about 7,000 m/sec, which is provided by the ignition of the second stage about 270 seconds after launch. The specific speed needed is determined by the orbit desired. Lower orbits require a smaller final speed (6,000 to 8,000 m/sec) than higher orbits (8,000 to 10,000 m/sec).

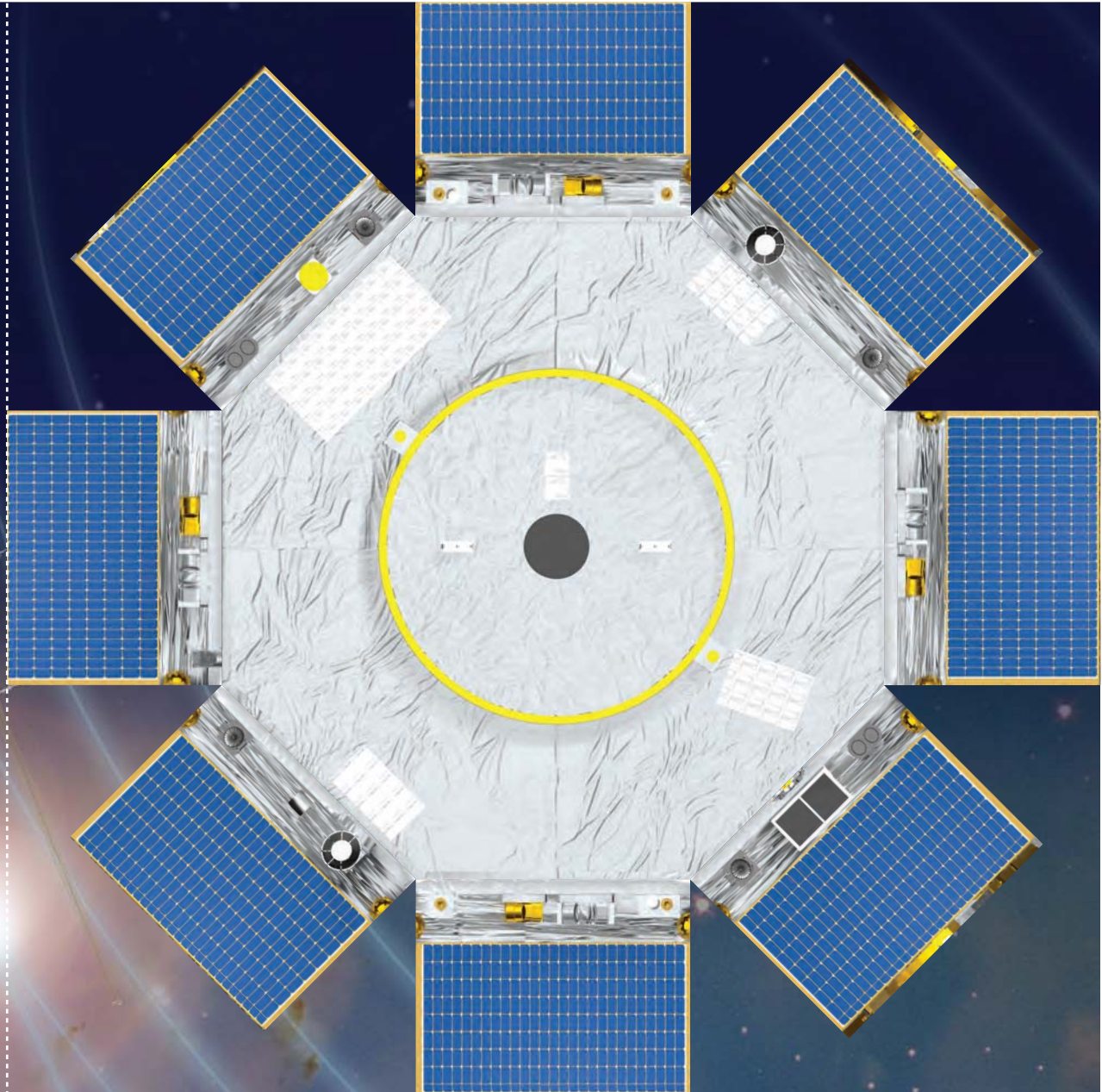


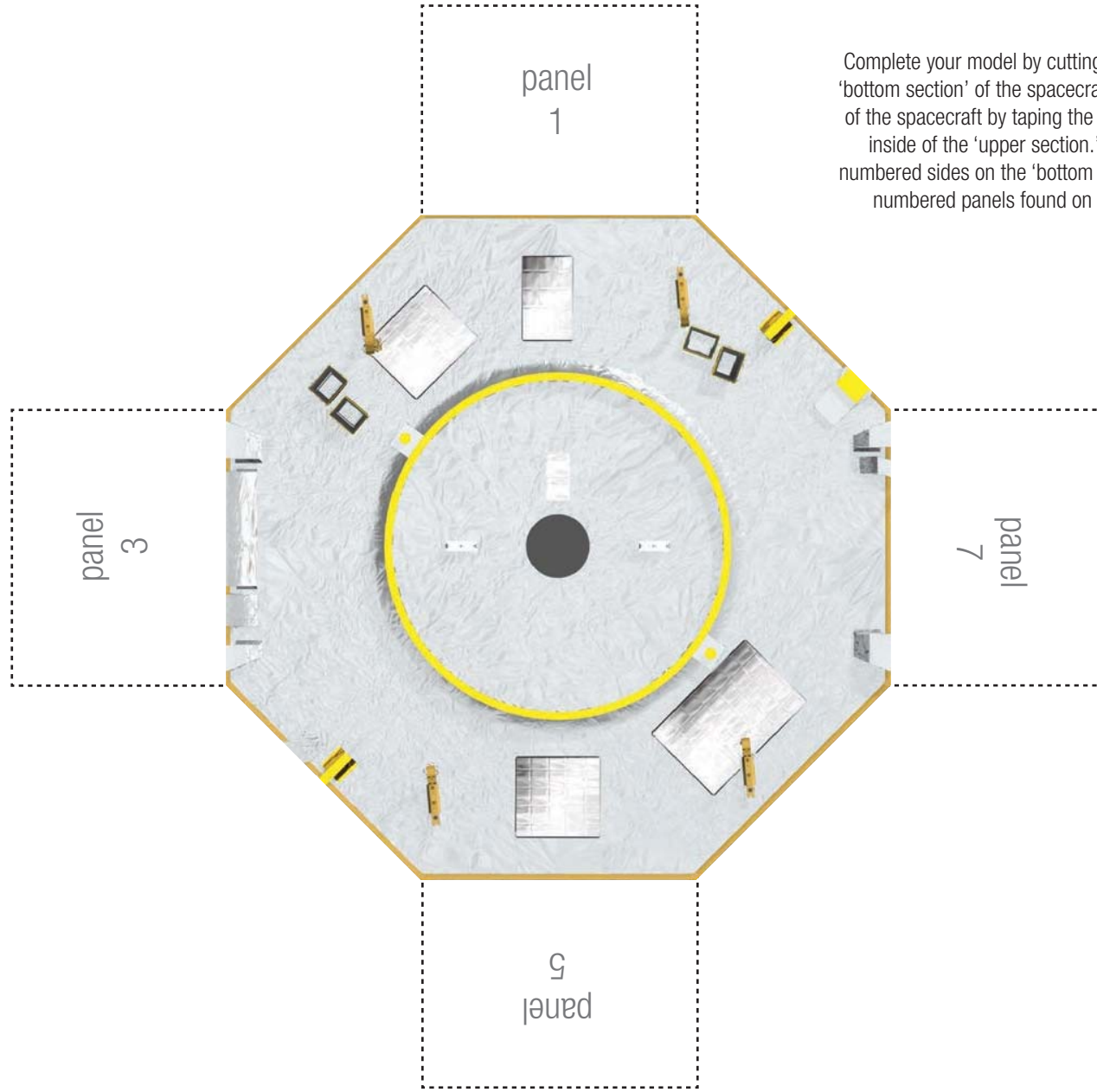
# MMS

Magnetospheric  
Multiscale  
Mission



National Aeronautics and  
Space Administration





Complete your model by cutting out and attaching this 'bottom section' of the spacecraft to the 'upper section' of the spacecraft by taping the four 'panel' tabs to the inside of the 'upper section.' Make sure that the numbered sides on the 'bottom section' match with the numbered panels found on the 'upper section.'

National Aeronautics and Space Administration

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