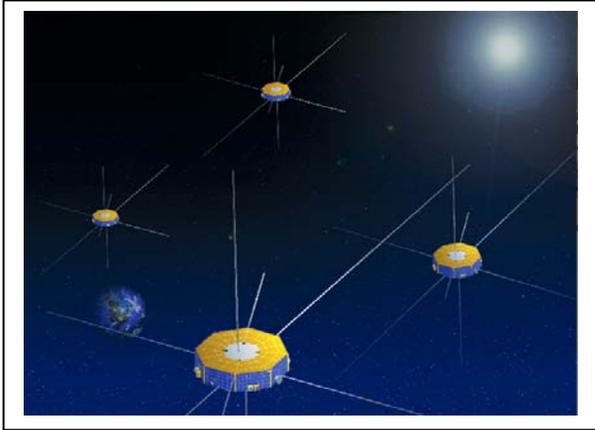


SpaceMath@NASA Supplemental Problems



In 2014, the four satellites of the Magnetosphere Multiscale (MMS) mission will be launched into orbit atop an Atlas-V421 rocket. These satellites, working together, will attempt to measure dynamic changes in Earth's magnetic field that physicists call magnetic reconnection events. These changes in the magnetic field are responsible for many different phenomena including Earth's polar aurora.

Soon after launch, the satellites will be placed into a Phase-1 elliptical orbit with Earth at one of the foci. The closest distance between the satellite and Earth, called perigee, will be at a distance of $1.2 R_e$, where $1.0 R_e$ equals the radius of Earth of 6,378 km. The farthest distance from Earth, called apogee, occurs at a distance of $12.0 R_e$. After a few years, the satellites will be moved into a Phase-2 elliptical orbit with an apogee of $25.0 R_e$ and a perigee of $1.2 R_e$.

The standard form for an ellipse is
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where x and y are the coordinates of a point on the ellipse, and the ellipse constants a and b , which are the semi-major and semi-minor axis dimensions of the ellipse. The relationship between the apogee (A) and perigee (P) distances, and the ellipse constants, a and b , and the eccentricity of the ellipse, e , are as follows:

$$A = a + c \quad P = a - c \quad e = c/a \quad b = a(1 - e^2)^{1/2}$$

Problem 1 – What are the equations for the semi-major and semi-minor axis, a and b , in terms of only A and P ?

Problem 2 – What are the equations of the MMS elliptical orbit in standard form, with all distances given in multiples of R_e for A) Phase-1 and B) Phase-2?

Problem 3 – What are the equations of the MMS elliptical orbit in the form $kx^2 + gy^2 = s$ where k , g and s are numerical constants rounded to integers for A) Phase-1 and B) Phase-2?

Answer Key

The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \quad e = c/a \quad b = a(1-e^2)^{1/2}$$

Problem 1 – What are the equations for the semi-major and semi-minor axis, a and b, in terms of only A and P?

Answer: Add the equations for A and P to get $A+P = 2a$, and $a = (A+P)/2$
Subtract the equations for A and P to get $A-P = 2c$, and $c = (A-P)/2$
Then by substitution $e = (A-P)/(A+P)$

And so
$$b = \frac{(A+P)}{2} \sqrt{1 - \frac{(A-P)^2}{(A+P)^2}} \quad \text{so} \quad b = \frac{1}{2} \sqrt{(A+P)^2 - (A-P)^2}$$

Expand and simplify:

$$b = \frac{1}{2} (A^2 + 2AP + P^2 - A^2 + 2AP - P^2)^{1/2}$$
$$b = \frac{1}{2} (4AP)^{1/2}$$
$$b = (AP)^{1/2}$$

Problem 2 –

Answer: A) for $A = 12.0$ and $P=1.2$, $a = (13.2/2) = 6.6 R_e$ $c = (12-1.2)/2 = 5.4 R_e$
 $e = (5.4/6.6) = 0.82$
 $b = (12 \times 1.2)^{1/2} = 3.8 R_e$

then
$$1 = \frac{x^2}{(6.6)^2} + \frac{y^2}{(3.8)^2}$$

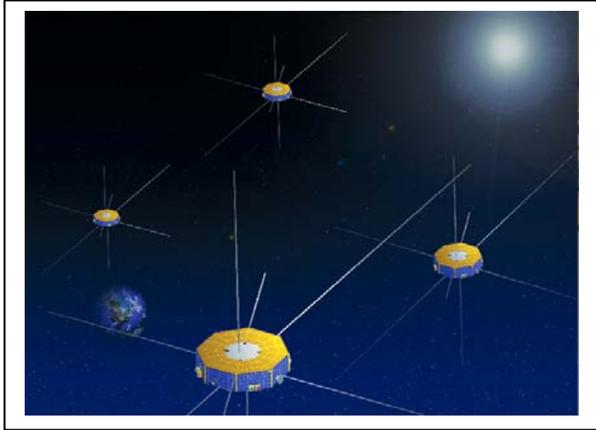
B) for $A = 25.0$ and $P=1.2$, $a = (26.2/2) = 13.1 R_e$ $c = (25-1.2)/2 = 11.9 R_e$
 $e = (11.9/13.1) = 0.91$
 $b = (25 \times 1.2)^{1/2} = 5.5 R_e$

then
$$1 = \frac{x^2}{(13.1)^2} + \frac{y^2}{(5.5)^2}$$

Problem 3 –

Answer: A) $1 = x^2/(6.6)^2 + y^2/(3.8)^2$ cross-multiply and simplify to get
 $(6.6)^2(3.8)^2 = (3.8)^2x^2 + (6.6)^2y^2$
 $662.55 = 14.4x^2 + 43.56y^2$ and rounded to the nearest integer:
 $663 = 14x^2 + 44y^2$

B) $1 = x^2/(13.1)^2 + y^2/(5.5)^2$ cross-multiply and simplify to get
 $(13.1)^2(5.5)^2 = (5.5)^2x^2 + (13.1)^2y^2$
 $5191.20 = 30.25x^2 + 171.61y^2$ and rounded to the nearest integer:
 $5191 = 30x^2 + 172y^2$



In 2014, the four satellites of the Magnetosphere Multiscale (MMS) mission will be launched into orbit atop an Atlas V421 rocket. These satellites, working together, will attempt to measure dynamic changes in Earth's magnetic field that physicists call magnetic reconnection events. These changes in the magnetic field are responsible for many different phenomena including Earth's polar aurora.

Soon after launch, the satellites will be placed into a Phase-1 elliptical orbit with Earth at one of the foci. After a few years, the satellites will be moved into a different 'Phase-2' elliptical orbit. The closest distance between the satellite and Earth is called the perigee. The farthest distance from Earth is called the apogee. The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \quad e = c/a \quad b = a(1 - e^2)^{1/2}$$

Where a , b and e are the semi-major and semi-minor axis lengths, and e is the eccentricity of the ellipse. The equations describing the Phase-1 and Phase-2 orbits are as shown below, with all distance units given in terms of multiples of 1 Earth radius ($1 R_E = 6,378 \text{ km}$):

$$\text{Phase-1} \quad 663 = 14x^2 + 44y^2$$

$$\text{Phase-2} \quad 5191 = 30x^2 + 172y^2$$

Problem 1 – In terms of kilometers, and to two significant figures, what is the semi-major axis distance, a , for the A) Phase-1 orbit? B) Phase-2 orbit?

Problem 2 – According to Kepler's Third Law, for orbits near Earth, the relationship between the semi-major axis distance, a , and the orbital period, T , is given by

$$a^3 = 287 T^2$$

where a is in units of R_E and T is in days. To the nearest tenth of a day, what are the estimated orbit periods for the MMS satellites in A) Phase-1? B) Phase-2?

Answer Key

The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \quad e = c/a \quad b = a(1-e^2)^{1/2}$$

The equations describing the Phase-1 and Phase-2 orbits are as shown below, with all distance units given in terms of multiples of 1 Earth radius ($1 R_e = 6,378 \text{ km}$):

$$\text{Phase-1} \quad 663 = 14x^2 + 44y^2$$

$$\text{Phase-2} \quad 5191 = 30x^2 + 172y^2$$

Problem 1 – In terms of kilometers, and to two significant figures, what is the semi-major axis distance, a , for the A) Phase-1 orbit? B) Phase-2 orbit?

Answer: A) Writing the Phase-1 equation in standard form:

$$1 = \frac{x^2}{(6.9)^2} + \frac{y^2}{(3.9)^2}$$

$$\begin{aligned} \text{then } a &= 6.9 R_e \\ &= 6.9 \times (6,378 \text{ km}) \\ &= 44,008 \text{ km.} \\ &= 44,000 \text{ km, to 2 SF} \end{aligned}$$

B) Writing the Phase-2 equation in standard form:

$$1 = \frac{x^2}{(13.1)^2} + \frac{y^2}{(5.5)^2}$$

$$\begin{aligned} \text{then } b &= 13.1 R_e \\ &= 13.1 \times (6,378 \text{ km}) \\ &= 83,552 \text{ km.} \\ &= 84,000 \text{ km, to 2 SF} \end{aligned}$$

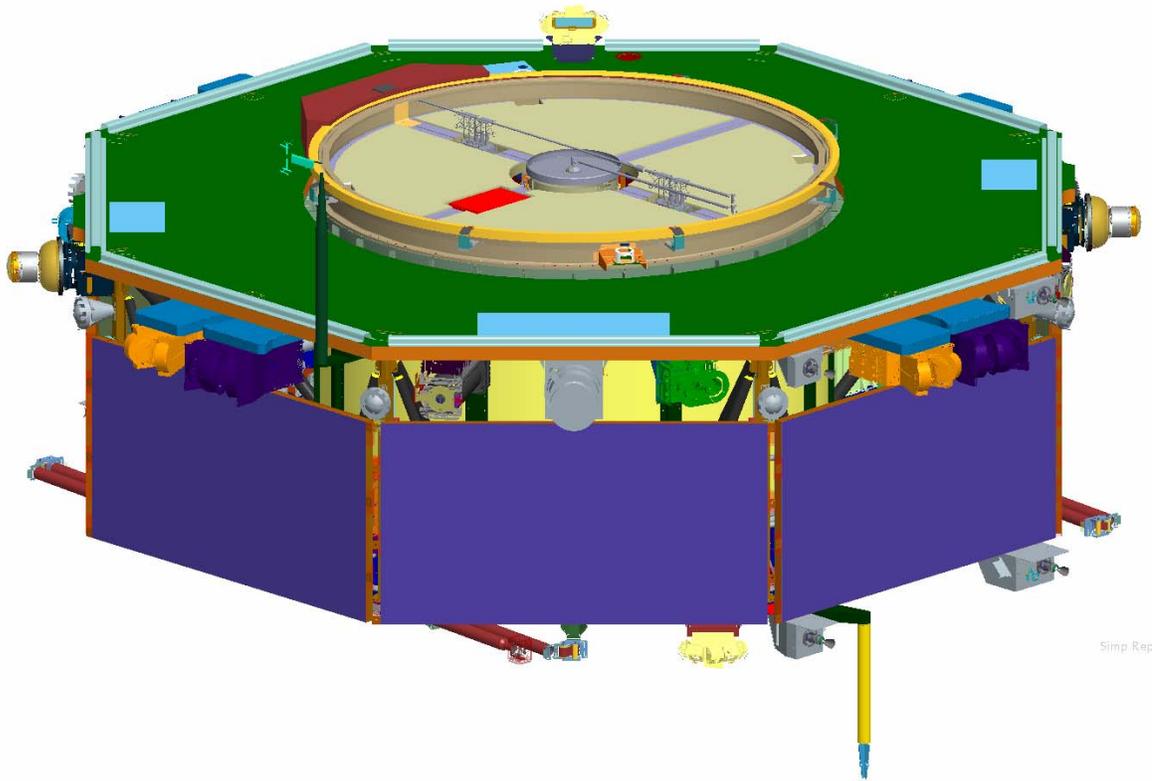
Problem 2 – According to Kepler's Third Law, for orbits near Earth, the relationship between the semi-major axis distance, a , and the orbital period, T , is given by

$$a^3 = 287 T^2$$

where a is in units of R_e and T is in days. To the nearest tenth of a day, what are the estimated orbit periods for the MMS satellites in A) Phase-1? B) Phase-2?

$$\text{Answer: A) } (6.9)^3 = 287 T^2, \text{ so for Phase-1, } T = 1.1 \text{ days.}$$

$$\text{B) } (13.1)^3 = 287 T^2, \text{ so for Phase-2, } T = 2.8 \text{ days.}$$



Each of the Magnetosphere Multiscale (MMS) satellites is in the shape of an octagonal prism. The faces of the satellite are partially covered with solar cells that will generate the electricity to operate the satellite and its many experiment modules.

Problem 1 - If the distance between opposite faces of the satellite is $D = 3.150$ meters, and the height of each solar panel is $h = 0.680$ meters, what is the width of one rectangular solar panel if $w = 0.414 D$?

Problem 2 - What is the surface area of one rectangular solar panel in the satellite?

Problem 3 If the solar cells generate 0.03 watts per square centimeter of surface area, how much power will be generated by a single face of the satellite?

Answer Key

Problem 1 - If the distance between opposite faces of the satellite is $D = 3.150$ meters, and the height of each solar panel is $h = 0.680$ meters, what is the width of one rectangular solar panel if $w = 0.414 D$?

Answer: The problem says that $D = 3.150$ meters, so $w = 0.414 (3.150) = \mathbf{1.30}$ meters.

Advanced students who know trigonometry can determine the width from the following geometry:

The relevant right-triangle has two sides that measure $D/2$ and $w/2$ with an angle of $45/2 = 22.5$. Then from trigonometry, $\tan(45/2) = w/D$. Since $\tan(22.5) = 0.414$, we have $w = 0.414 (D)$, and so $\mathbf{w = 1.30}$ meters.

Problem 2 - The area of each solar panel is just $A = (1.30 \text{ meters}) \times (0.680 \text{ meters})$ so

$$\mathbf{Area = 0.88 \text{ meter}^2}.$$

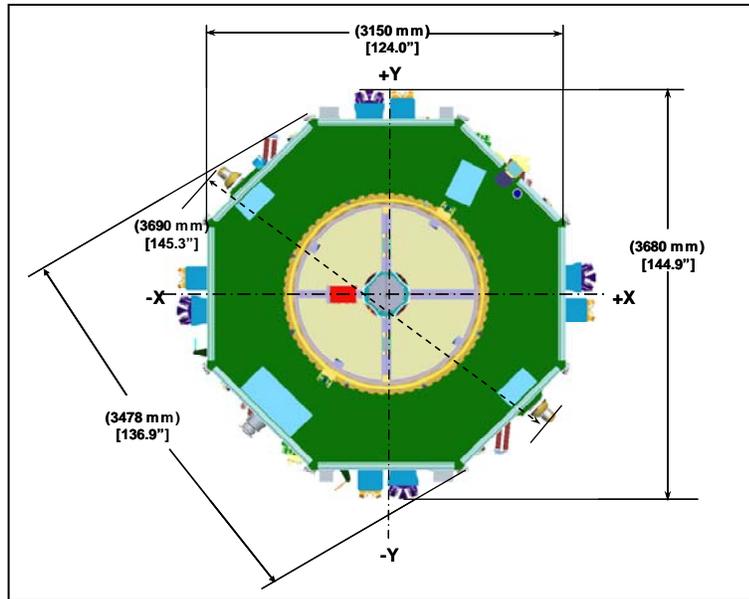
Problem 3 - If the solar cells generate 0.03 watts per square centimeter of surface area, how much power will be generated by a single face of the satellite?

Answer: A single face of the satellite has a solar panel with an area of 0.88 meters^2 . First convert this into square centimeters

$$\begin{aligned} A &= 0.88 \text{ meter}^2 \times (100\text{cm} / 1 \text{ m}) \times (100 \text{ cm}/1 \text{ m}) \\ &= 8800 \text{ cm}^2 \end{aligned}$$

Then multiply by the solar power area factor of 0.03 watts/cm^2 to get

$$\mathbf{Power = 264 \text{ watts.}}$$



The Magnetosphere Multiscale (MMS) constellation consists of 4 identical satellites that will be launched into orbit in 2014 to investigate Earth's magnetic field in space. Each satellite is an octagonal prism with a face-to-face diameter of 3.15 meters and a height of 0.896 meters. A central, cylindrical hole has been cut out of each satellite to accommodate the steering rockets and fuel tanks. This cylindrical hole has a diameter of 1.66 meters.

Problem 1 - What is the formula for the area of an octagon with a diameter of D meters, if the area of one of the 16 inscribed right triangles is given by the formula $A = 0.104D^2$?

Problem 2 - To three significant figures, what is the total surface area, including side faces, of a single MMS satellite? (Hint: don't forget the cylindrical hole!)

Problem 1 - What is the formula for the surface area of an octagon with a diameter of D meters if the area of one of the 16 inscribed right triangles is given by the formula $A = 0.104D^2$?

Answer: Note: Draw an octagon with the stated dimensions. Reduce the octagonal area to the areas of 16 right-triangles with sides w and D/2. Using trigonometry, $w/2 = D/2 \tan(45/2)$ so $w = 0.414 D$, and then the area of a single triangle is $A = 1/2 (D/2) \times (0.414D)$ or $A = 0.104 D^2$.

The total area of a single octagonal face is then $A = 16 \times (0.104) D^2$, or **$A = 1.664D^2$** .

Problem 2 - To three significant figures, what is the total surface area, including side faces, of a single MMS satellite?

Answer: The surfaces consist of 2 octagons, and 8 rectangular side panels. The total area is then $A = 2 (1.664D^2) + 8 (h)(w)$. For $D = 3.15$ meters, $h = 0.896$ meters and $w = 0.414(3.15) = 1.30$ meters, we have a total area of

$A = 2(1.664)(3.15)^2 + 8(0.896)(1.30) = 42.34 \text{ meters}^2$. This is for a solid, regular octagonal cylinder. However, for an MMS satellite, a cylindrical hole has been removed. This means that for the top and bottom faces, a circular area of $A = 2 \times \pi (1.66/2)^2 = 4.33 \text{ meters}^2$ has been removed, and a surface area for the inside cylindrical hole has been ADDED. This surface area is just $A = 2 \pi r h$ or $A = 2 (3.14) (1.66/2)(0.896) = 4.67 \text{ meters}^2$. So the total area is just

$$A = 42.34 \text{ meters}^2 - 4.33 \text{ meters}^2 + 4.67 \text{ meters}^2$$

$$A = 42.68 \text{ meters}^2, \text{ which to three SF is just } \mathbf{A = 42.7 \text{ meters}^2}.$$

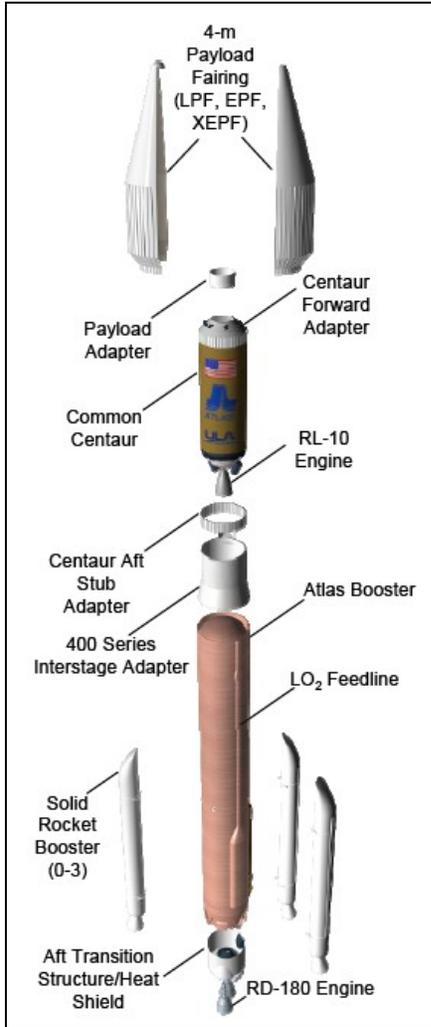
Problem 3 - To three significant figures, what is the volume of a single MMS satellite?

Answer: The surface area of a single satellite hexagonal face is $A = 1.664D^2$ and the volume is just $V = 1.664D^2 h$. With a central cylindrical volume subtracted with $V_c = \pi r^2 h$, then $V = h (1.664D^2 - \pi r^2)$. For a single MMS satellite

$$V = (0.896)(1.664(3.15)^2 - 3.141(1.66/2)^2)$$

$$V = 14.79 - 1.94$$

$$V = 12.85 \text{ meters}^3. \text{ so to three SF we have } \mathbf{V = 12.9 \text{ meters}^3}.$$



The Magnetosphere Multiscale (MMS) satellite constellation will be launched into orbit in 2014 atop an Atlas V421XEPF rocket. There are dozens of different configurations of Atlas rockets depending on the mass and orbit destination of the payload being launched. One of these that is similar to the MMS rocket is shown in the figure to the left. The Atlas V421 consists of single Atlas rocket booster with two strap-on solid rocket boosters, and a second-stage Centaur rocket booster with the MMS satellite stack attached above the Centaur.

Atlas: Height = 32.46 meters
 Diameter = 3.81 meters
 Mass = 399,700 kg

Centaur: Height = 12.68 meters
 Diameter = 3.05 meters
 Mass = 23,100 kg

Payload: Height = 13.81 meters
 Diameter = 4.2 meters
 Mass = 7,487 kg

Problem 1 - What is the total length of the MMS launch vehicle in A) meters? B) feet? C) inches? (Note: 1 meter = 3.281 feet)

Problem 2 - To the nearest tenth of a percent, what percentage of the launch vehicle height is occupied by the payload?

Problem 3 - To the nearest tenth of a percent, if each satellite has a mass of 1,250 kg, what percentage of the entire rocket mass is occupied by the four satellites?



Answer Key

Atlas: Height = 32.46 meters
Diameter = 3.81 meters Mass = 399,700 kg

Centaur: Height = 12.68 meters
Diameter = 3.05 meters Mass = 23,100 kg

Payload: Height = 13.81 meters
Diameter = 4.2 meters Mass = 7,487 kg

Problem 1 - What is the total length of the MMS launch vehicle in A) meters? B) feet? C) Inches?

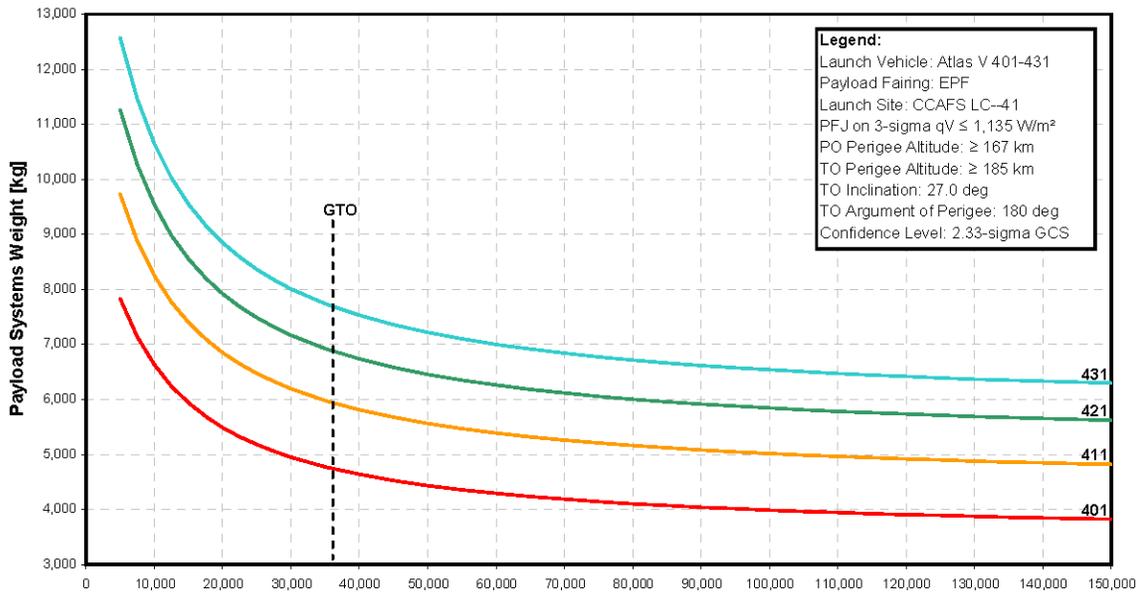
Answer: A) $H = 32.46 \text{ meters} + 12.68 \text{ meters} + 13.81 \text{ meters} = \mathbf{58.95 \text{ meters}}$.
B) $H = 58.95 \text{ meters} \times (3.281 \text{ feet/meters}) = \mathbf{193.41 \text{ feet}}$.
C) $H = 193.41 \text{ feet} \times (12 \text{ inches} / 1 \text{ foot}) = \mathbf{2320.92 \text{ inches}}$.

Problem 2 - To the nearest tenth of a percent, what percentage of the launch vehicle height is occupied by the payload?

Answer: Total length = 58.95 meters. Payload = 13.81 meters so
 $P = 100\% (13.81/58.95) = \mathbf{23.4\%}$

Problem 3 - To the nearest tenth of a percent, if each satellite has a mass of 1,250 kg, what percentage of the entire rocket mass is occupied by the four satellites?

Answer: The combined masses for the Atlas booster, the Centaur upper stage and the payload is $M = 399,700 \text{ kg} + 23,100 \text{ kg} + 2,487 \text{ kg} + 4(1,250 \text{ kg}) = 430,287 \text{ kg}$. The mass in the satellites is $m = 4(1,250 \text{ kg}) = 5000 \text{ kg}$, so the percentage is just
 $P = 100\% (5000/430287) = \mathbf{1.2\%}$.



This figure, obtained from the Boeing Corporation ‘Atlas V Users Guide: 2010’ shows the maximum payload mass that can be launched into a range of orbits with apogees from 6,378 km to 150,000 km. The apogee distance is the maximum distance from the center of Earth that the orbit will take the payload. The specific models of Atlas rocket are given on the right-hand edge and include the Atlas V401, V411, V421 and V431. For example, the Atlas V401 can lift any payload with a mass of less than 4,000 kg into an orbit with an apogee of no more than 100,000 km.

Problem 1 – For a particular Atlas V model, what does the curve for that model tell you about payload mass and maximum altitude?

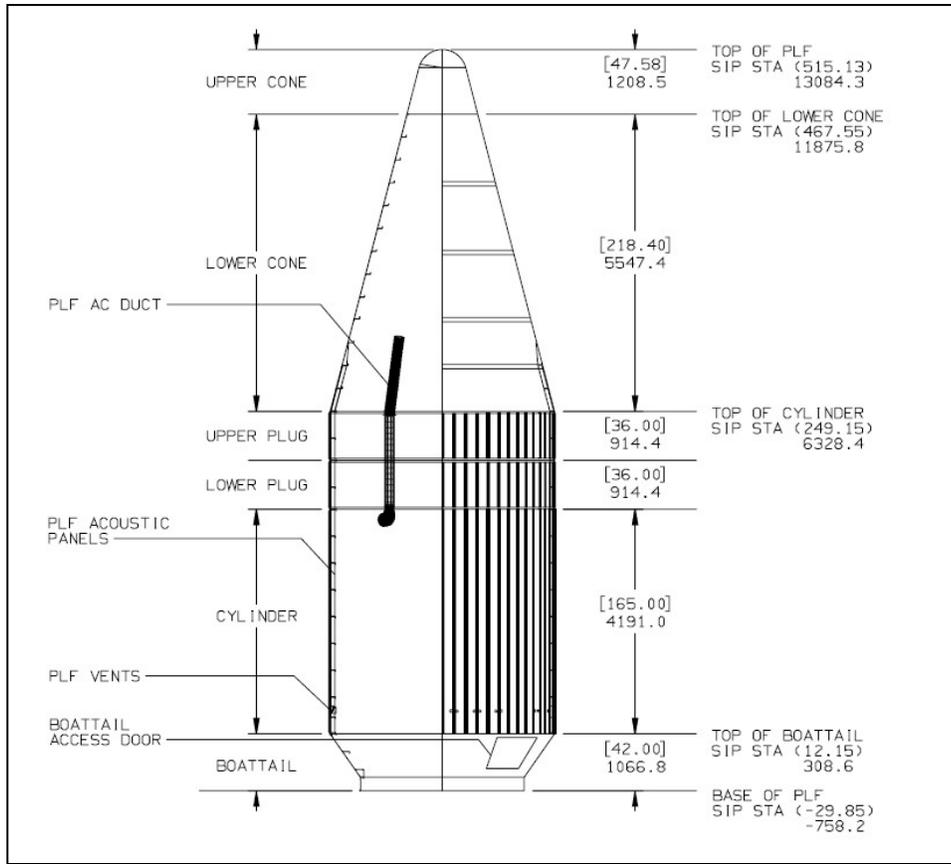
Problem 2 – Suppose a scientist wanted to place a 6,500 kg research satellite into orbit with an apogee of 70,000 km. What is the best choice of launch vehicle to satisfy this requirement?

Problem 1 – For a particular Atlas V model, what does the curve for that model tell you about payload mass and maximum altitude?

Answer: The curves all decline in payload mass as the orbit apogee increases. This means that there is a trade-off between the mass you can place into orbit and the maximum apogee for that mass. The less mass your payload has, the greater is the apogee of the orbit that you can reach. This is the same concept as it is easier for you to throw a 3 ounce tennis ball to a higher altitude than a 2-pound lead weight! The same relationship works for rockets.

Problem 2 – Suppose a scientist wanted to place a 6,500 kg research satellite into orbit with an apogee of 70,000 km. What is the best choice of launch vehicle to satisfy this requirement?

Answer: We find '70,000 km' on the horizontal axis and draw a vertical line. Next we draw a horizontal line from '6,500 kg' on the vertical axis until it intercepts our vertical line at '70,000 km'. The intersection point lies just above the curve for the V 421, meaning that it is not powerful enough for the task, but it lies below the curve for the **Atlas V431** launch vehicle, so this vehicle has the capacity to launch this payload to the indicated apogee.



The four satellites of the Magnetosphere Multiscale (MMS) mission will be stacked vertically inside a nose-cone payload shroud, which will be jettisoned when the payload reaches orbit. The diagram above shows the dimensions of the payload. The bracketed numbers are in inches. The numbers below the brackets are the corresponding dimensions in millimeters. The diameter of the cylindrical section is 4.2 meters. The cylindrical payload section is topped by a conical ‘nose-cone’ section, which in turn comes to an end in a hemispherical cap with a diameter of 910 millimeters.

Problem 1 – To 2 significant figures, what is the volume of the cylindrical payload section including the Upper and Lower Plugs; A) in cubic meters? B) in cubic centimeters? C) in cubic feet? (1 foot = 30.48 cm).

Problem 2 – To 2 significant figures, what is the volume for the hemispherical cap at the top of the nose-cone in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

Problem 3 – The formula for the volume of a truncated right circular cone is given by $V = \frac{1}{3} \pi (R^2 + rR + r^2)h$, where R is the radius at the base, and r is the radius at the truncation. To 2 significant figures, what is the volume of the conical section in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

Problem 1 – To 2 significant figures, what is the volume of the cylindrical payload section including the Upper and Lower Plugs A) in cubic meters? B) in cubic centimeters? C) in cubic feet? (1 foot = 30.48 cm)

Answer: A) $r = 4.2/2 = 2.1$ meters, and converting the measurements in millimeters to meters, $h = 4.191\text{m} + 0.914\text{m} + 0.914\text{m} = 6.019\text{m}$,
so $V = \pi r^2 h = 3.141 (2.100)^2 (6.019) = \mathbf{83 \text{ meters}^3}$.

B) $V = 83.0 \text{ m}^3 \times (100 \text{ cm} / 1\text{m})^3 = \mathbf{83,000,000 \text{ cm}^3}$.

C) $V = 83000000 \text{ cm}^3 \times (1 \text{ foot} / 30.48 \text{ cm})^3 = \mathbf{2,900 \text{ feet}^3}$.

Problem 2 – To 2 significant figures, what is the volume for the hemispherical cap at the top of the nose-cone in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

Answer: A) $V = 2/3 \pi R^3$ and $R = 0.91 \text{ m}/2 = 0.455 \text{ m}$, so $V = \mathbf{0.20 \text{ meters}^3}$.

B) $V = 0.20 \text{ m}^3 \times (100 \text{ cm}/1 \text{ m})^3 = \mathbf{200,000 \text{ cm}^3}$

C) $V = 200,000 \text{ cm}^3 \times (1 \text{ foot}/30.48 \text{ cm})^3 = \mathbf{7.0 \text{ feet}^3}$

Problem 3 – The formula for the volume of a truncated circular cone is given by $V = 1/3 \pi (R^2 + rR + r^2)$, where R is the radius at the base, and r is the radius at the truncation. To 2 significant figures, what is the volume of the conical section in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

Answer: A) $R = 4.2 \text{ m}/2 = 2.1$ meters, and $r = 0.910 \text{ m}/2 = 0.455 \text{ m}$, so
 $V = 0.333 (3.141) (2.1^2 + (2.1)(0.455) + 0.455^2) = \mathbf{5.8 \text{ meters}^3}$.

B) $V = 5.8 \text{ m}^3 \times (100 \text{ cm} / 1 \text{ m})^3 = \mathbf{5,800,000 \text{ cm}^3}$.

C) $V = 5,800,000 \text{ cm}^3 \times (1 \text{ foot} / 30.48 \text{ cm})^3 = \mathbf{210 \text{ feet}^3}$.



The Magnetosphere Multiscale (MMS) mission will be launched from Pad 41 at the Cape Canaveral US Air Force Launch Facility in Florida in 2014. The image above shows a spectacular satellite view of this complex. The short, horizontal line to the lower left indicates a length of 100 meters.

Problem 1 – With the help of a millimeter ruler, what is the scale of this image in meters per millimeter?

Problem 2 – The circular road is centered on the location of the launch pad for the Atlas V rocket. What is the circumference of this road to the nearest meter?

Problem 3 – At a comfortable walking pace of 1.5 meter/sec, to the nearest minute, how long would it take you to walk around this perimeter road?

Problem 4 – There are four towers surrounding the launch pad, and their shadows can be seen pointing to the upper left. If a tower is 73 meters tall, create a scale model showing the horizontal shadow length and the vertical tower height, and determine the angle of the sun at the time the image was made.

Answer Key

Problem 1 – With the help of a millimeter ruler, what is the scale of this image in meters per millimeter?

Answer: For ordinary reproduction scales, students should measure the '100 meter' bar to be 12 mm long, so the scale of the image is $100 \text{ meters}/12 \text{ mm} = \mathbf{8.3 \text{ meters/mm}}$.

Problem 2 – The circular road is centered on the location of the launch gantry for the Atlas-V rocket. What is the circumference of this road to the nearest meter?

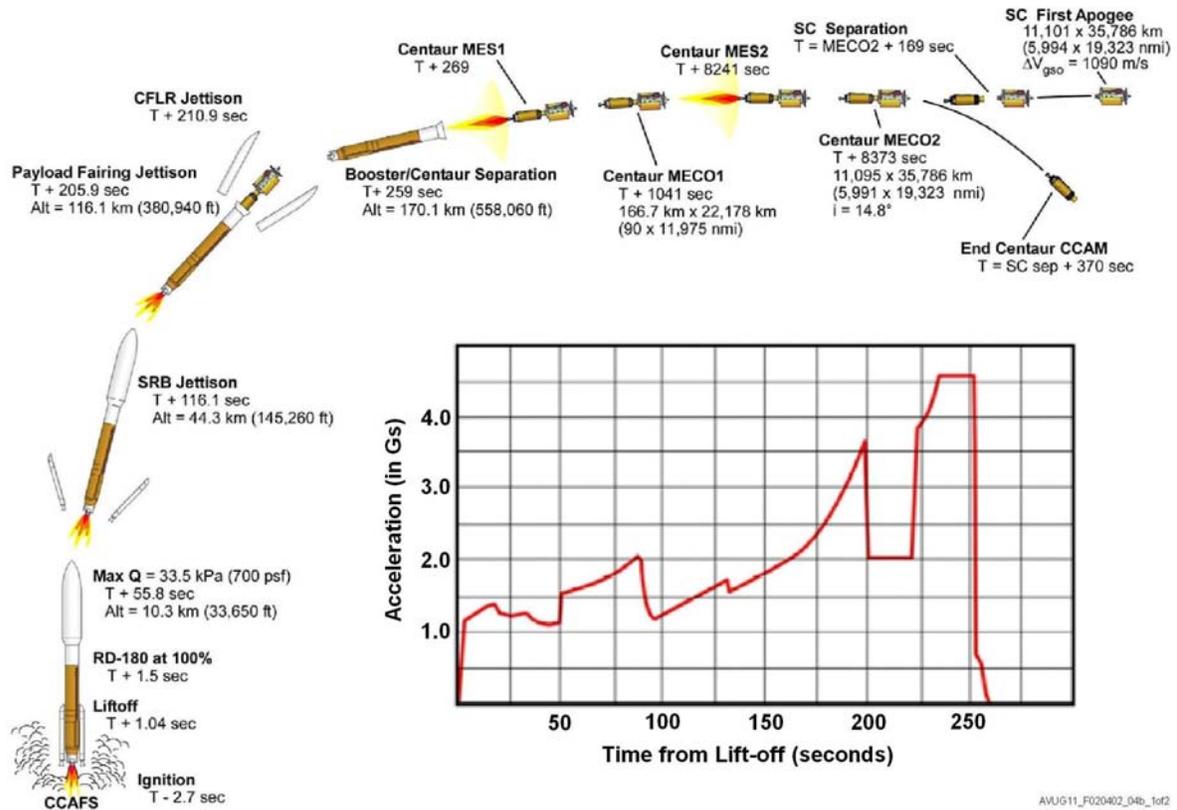
Answer: The diameter of the road is 41 mm or 340 meters. The circumference is $C = 3.141 \times 340 \text{ meters} = \mathbf{1068 \text{ meters}}$.

Problem 3 – At a comfortable walking pace of 1.5 meter/sec, to the nearest minute, how long would it take you to walk around this perimeter road?

Answer: $T = \text{distance}/\text{speed}$ so
 $T = 1068 \text{ meters}/1.5 = 712 \text{ seconds}$ or **12 minutes**.

Problem 4 – There are four towers surrounding the launch gantry, and their shadows can be seen pointing to the upper left. If a tower is 73 meters tall, create a scale model showing the horizontal shadow length and the vertical tower height, and determine the angle of the sun at the time the image was made.

Answer: The shadow of one of the towers measures about 17 mm or 141 meters. The two sides of the right-triangle are therefore 73 meters and 141 meters. Students may draw a scaled model of this triangle, then use a protractor to measure the sun elevation angle opposite the '73 meter' segment. Or they may use $\tan(\theta) = 73 \text{ meters}/141 \text{ meters}$ and so $\theta = \mathbf{27 \text{ degrees}}$.



This diagram, provided by the Boeing *Atlas V Launch Services User's Guide* shows the major events during the launch of an Atlas V521 rocket, which is similar to the rocket planned for the MMS launch in 2014. The figure shows the events in the V521 timeline starting from -2.7 seconds before launch through the Centaur rocket main engine cut off 'MECO2' event at 8,373 seconds after launch. The graph shows the acceleration of the payload during the first 250 seconds after launch. Acceleration is given in Earth gravities, where 1.0 G = 9.8 meters/sec².

Problem 1 – To the nearest tenth of a G, what is the acceleration at the time of:

- Maximum aerodynamic pressure, called Max-Q ?
- Solid rocket booster (SRB) jettison?
- Payload fairing jettison?
- Booster/Centaur separation?

Advanced Math Challenge: The speed of the rocket at a particular time, T, is the area under the acceleration curve (in meters/sec²) from the time of launch to the time, T. By approximating the areas as combinations of rectangles and triangles, and rounding your final answers to two significant figures, about what is the rocket speed at a time of:

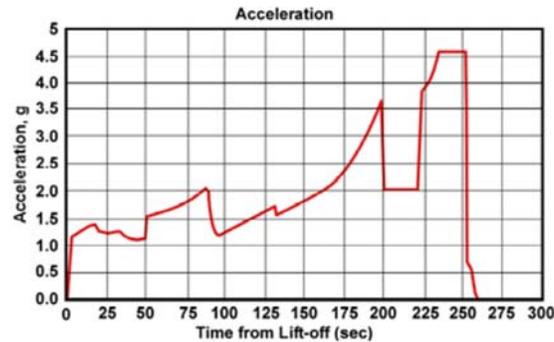
- T = 50 seconds?
- T = 100 seconds?
- T = 200 seconds?
- T = 250 seconds?

Answer Key

Problem 1 – To the nearest tenth of a G, what is the acceleration at the time of:

Answer:

- | | |
|--|---------------------------------------|
| A) Maximum aerodynamic pressure, called Max-Q? | A = 1.6 Gs |
| B) Solid Rocket Booster (SRB) jettison? | A = 1.5 Gs |
| C) Payload Fairing Jettison? | A = 2.0 Gs |
| D) Booster/Centaur Separation? | A = 0.0 Gs (thrust no longer applied) |



Advanced Math Challenge – The speed of the rocket at a particular time, T, is the area under the acceleration curve (in meters/sec²) from the time of launch to the time, T. By approximating the areas as combinations of rectangles and triangles, and rounding your final answers to two significant figures, about what is the rocket speed at a time of:

- A) T = 50 seconds?

$$V = (50 \text{ seconds}) (1.3\text{Gs}) (9.8 \text{ m/s}^2) = \mathbf{640 \text{ meters/sec.}}$$

- B) T = 100 seconds?

$$\begin{aligned} V &= 640 \text{ meters/sec} + (50 \text{ sec})(1.5\text{Gs})(9.8\text{m/s}^2) + 1/2(50\text{sec})(2.0-1.5)(9.8\text{m/s}^2) \\ &= 640 \text{ meters/sec} + 735 \text{ m/sec} + 123 \text{ m/sec} \\ &= \mathbf{1500 \text{ meters/sec}} \end{aligned}$$

- C) T = 200 seconds?

$$\begin{aligned} V &= 1500 \text{ meters/sec} + (200-100)(1.2 \text{ Gs})(9.8 \text{ m/s}^2) + \frac{1}{2} (200-100)(3.6-1.2)(9.8\text{m/s}^2) \\ &= 1500 \text{ m/sec} + 1200 \text{ m/sec} + 1200 \text{ m/sec} \\ &= \mathbf{3900 \text{ meters/sec.}} \end{aligned}$$

- D) T = 250 seconds?

$$\begin{aligned} V &= 3900 \text{ meters/sec} + (225-200)(2.0\text{Gs})(9.8 \text{ m/s}^2) + (250-225)(4.5 \text{ Gs})(9.8 \text{ m/s}^2) \\ &= 3900 \text{ m/sec} + 490 \text{ m/s} + 1100 \text{ m/s} \\ &= \mathbf{5500 \text{ meters/sec.}} \end{aligned}$$

Note: Students answers will vary. The biggest challenge is to convert Gs to meters/sec² to get physical units in terms of meters and time.

To reach 'orbit', the payload needs a speed of about 7,000 m/sec, which is provided by the ignition of the second stage about 270 seconds after launch. The specific speed needed is determined by the orbit desired. Lower orbits require a smaller final speed (6,000 to 8,000 m/sec) than higher orbits (8,000 to 10,000 m/sec).